Following the equilibrium path during crack propagation under quasi-static loads: linear elastic fracture mechanics and phase-field approaches

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ABSTRACT

The equilibrium path [1] of an elastic body with an initial crack represents the sequence of states (displacement and crack geometry) at which crack propagation threshold is met under a specific quasi-static proportional loading sequence. Constructing this equilibrium path requires adjusting the load amplitude to maintain the crack at its propagation threshold throughout its progression. Following the equilibrium path gives the stable and incremental crack propagation, at the cost of neglecting dynamic effects arising during instabilities. Following the equilibrium path in numerical simulations (e.g., phase-field fracture models) ensures controlled and incremental crack propagation so that each crack increment minimizes the energy (in contrast to minimizing the energy over the entire crack path).

This work proposes methodologies to determine the equilibrium path with the frameworks of Linear Elastic Fracture Mechanics (LEFM) and phase-field fracture models. It focuses on the case of a single controlled load, but extensions to more complex loads will also be discussed.

We first examine the analytical case of a finite elastic body with a known crack path. This simple example introduces the core principles of path-following methods in fracture mechanics, serving as a foundation for more complex scenarios. We then address cases where the crack path is unknown a priori, requiring adaptations of the methodology to integrate numerical tools for elastic solutions, Stress Intensity Factor (SIF) computation, and crack propagation solution. Finally, we present the determination of the equilibrium path using phase-field fracture models, employing an extension [2] of arc-length methods.

REFERENCES

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