Influence of the mesh on the crack path in phase-field fracture simulations

GAMM PF 25 and Materials/Microstructure modelling

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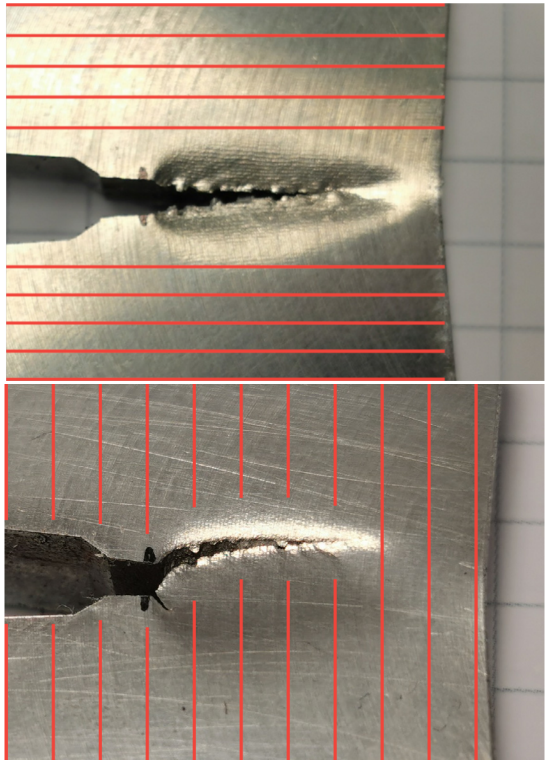
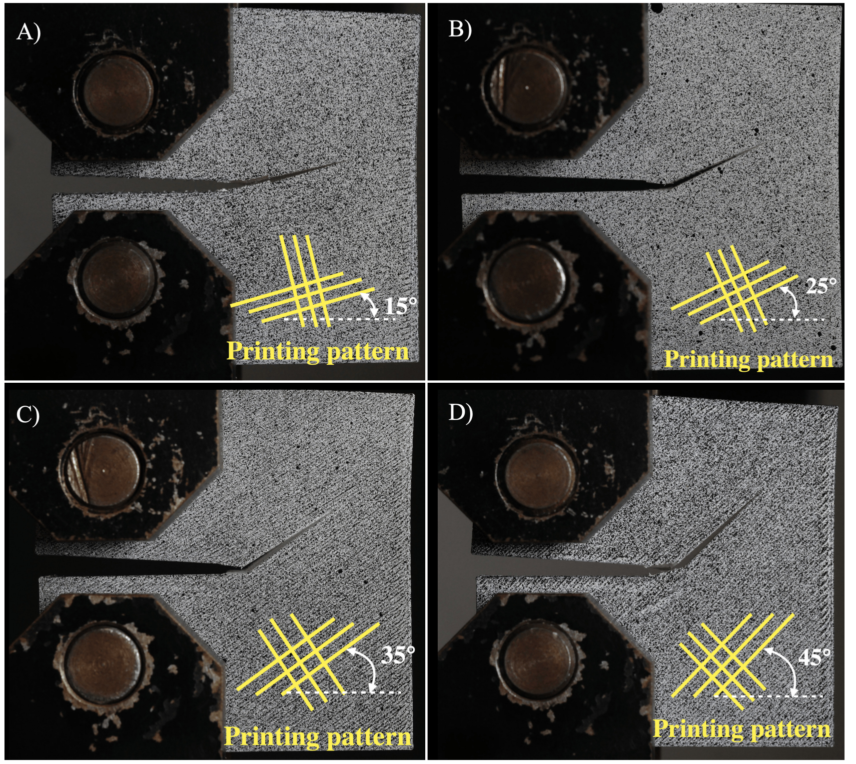
Crack propagation in 3D-printed structures



Duplex stainless steel by DED (Roucou et al., 2023)



Polycarbonate CT specimen by FDM (Zhai, 2023)

Global objective

Modelling and simulating quasi-static crack propagation in 3D-printed structures

The problem we want to solve

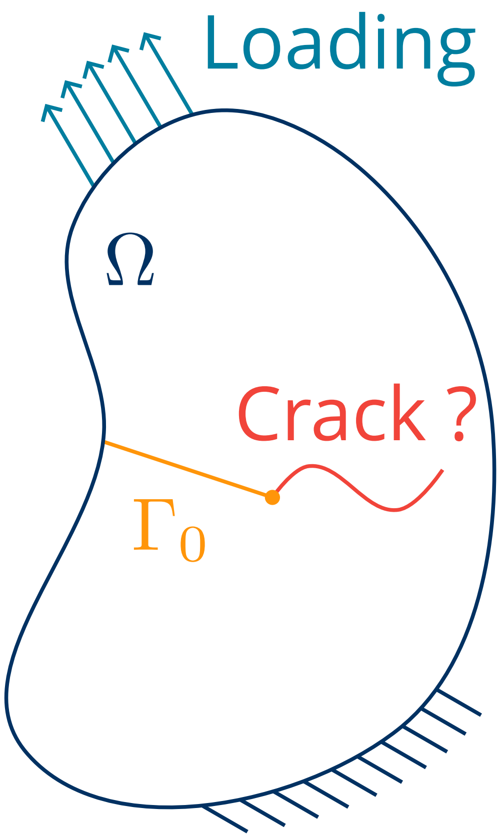
Fracture mechanics

We consider

- a domain Ω with a crack Γ_0,
- an elastic material (E, ν),
- a force and/or displacement load,

and we want to determine

- the crack path,
- the evolution of the displacement field.



To solve this problem, we want to employ numerical methods.

Linear Elastic Fracture Mechanics (LEFM)

State

The state of a domain Ω is described by:

- the displacement field $\boldsymbol{u}(\boldsymbol{x})$,
- the crack length a.

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The crack propagates when:

$$\underset{\text{elastic energy release}}{G} = \underset{\text{fracture energy required}}{G_c}$$

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Variational approach to fracture (Francfort & Marigo, 1998)

The state minimizes the potential energy $\mathcal{P}$,

$$(\boldsymbol{u}, a) = \arg\min_{\substack{\boldsymbol{u}' \in \mathcal{U}\\ a' \in \mathcal{A}}} \mathcal{P}(\boldsymbol{u}, a), \quad \mathcal{P}(\boldsymbol{u}, a) = \underbrace{\mathcal{E}(\boldsymbol{u}', a')}_{\text{elastic}} + \underbrace{\mathcal{D}(a')}_{\text{dissipation}} - \underbrace{\mathcal{W}_{\text{ext}}(\boldsymbol{u}')}_{\text{external work}}.$$

with $\quad \mathcal{D}(a) = \int_{\Gamma(a)} G_c \, \mathrm{d}S.$

Variational Phase-Field Model for fracture (VPFM)

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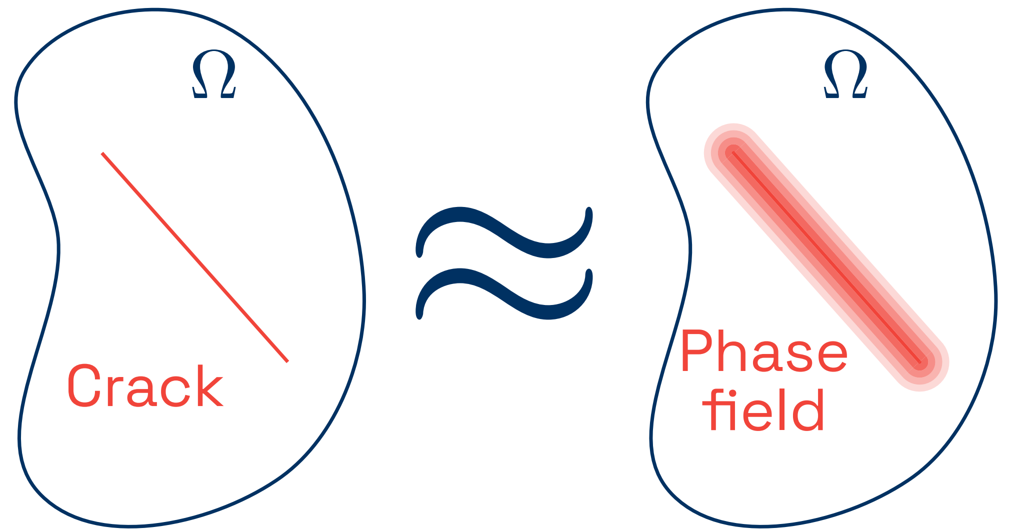
- the displacement field $\boldsymbol{u}(\boldsymbol{x})$,
- the crack phase field $\alpha(\boldsymbol{x})$,
  - $\alpha = 0 \rightarrow \text{unbroken}$,
  - $\alpha = 1 \rightarrow \text{broken}$,
  - $\alpha(t + \Delta t) \geq \alpha(t)$.



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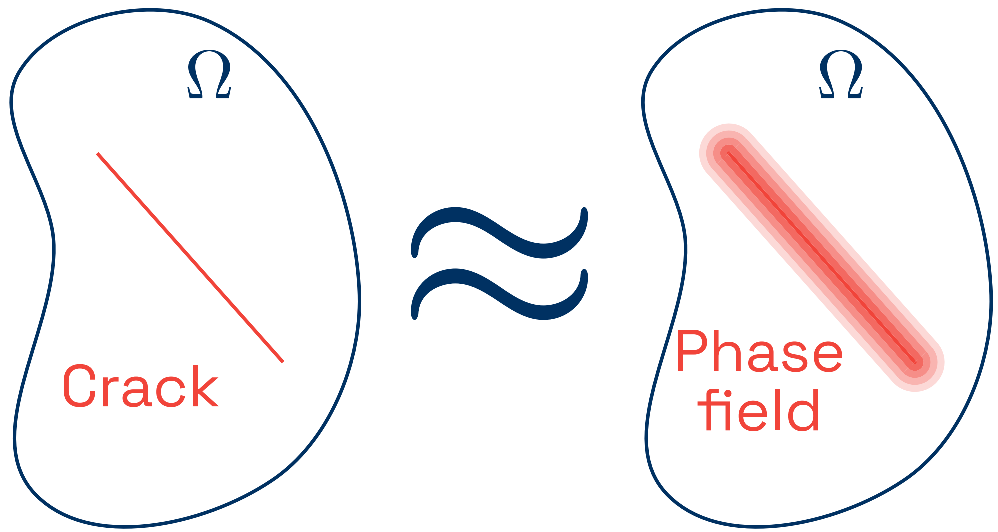

Variational phase field model (Bourdin et al., 2000; Francfort & Marigo, 1998)

The state minimizes the regularized potential energy $\mathcal{P}$,

$$(\boldsymbol{u}, \alpha) = \arg \min_{\substack{\boldsymbol{u}' \in \mathcal{U} \\ \alpha' \in \mathcal{A}}} \mathcal{P}(\boldsymbol{u}, \alpha), \quad \mathcal{P}(\boldsymbol{u}, \alpha) = \underbrace{\mathcal{E}(\boldsymbol{u}', \alpha')}_{\text{elastic}} + \underbrace{\mathcal{D}(\alpha')}_{\text{dissipation}} - \underbrace{\mathcal{W}_{\text{ext}}(\boldsymbol{u}')}_{\text{external work}} .$$

Γ-convergence of VPFM towards LEFM

We consider the classic dissipation functional

$$\mathcal{D}(\alpha) = \frac{G_0}{c_w} \int_\Omega \frac{w(\alpha)}{\ell} + \ell |\nabla \alpha|^2 \, \mathrm{d}x.$$

With **continous** field α (Braides, 1998; Giacomini, 2005),

$$\mathcal{D}(\alpha) \underset{\ell \to 0}{\to} \int_\Gamma G_0 \, \mathrm{d}S.$$

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$$\mathcal{D}(\alpha) \underset{\ell \to 0}{\rightarrow} \int_\Gamma G_0 \, \mathrm{d}S.$$

However, with a **discrete** field α (Negri, 1999, 2003),

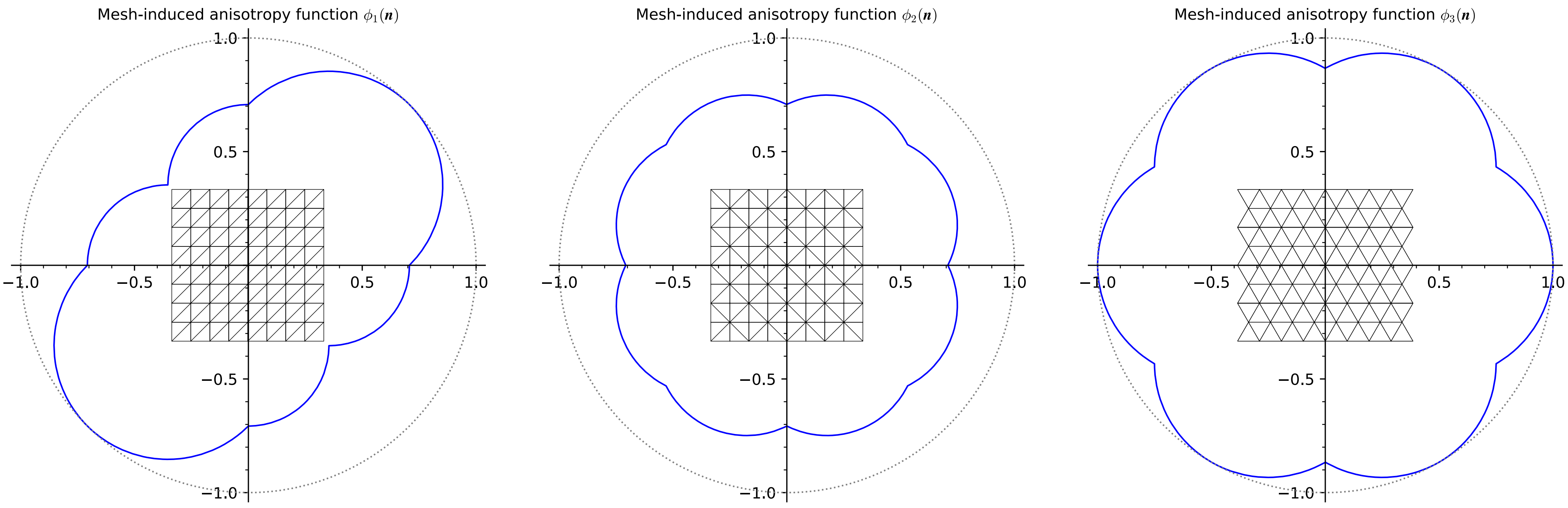
$$\mathcal{D}(\alpha) \underset{\ell \to 0}{\rightarrow} \int_\Gamma G_0 \, \varphi(\theta) \, \mathrm{d}S.$$

Observation

The discretization induces **artificial anisotropy**: $G_0 \, \phi(\theta) = G_c(\theta)$.

Illustration of mesh-induced anisotropy

Negri (1999), Negri (2003)



Polar plot of the mesh-induced anisotropy function $\phi(\boldsymbol{n})$ for different triangulation (based on the calculations of Negri (2003)).

Numeric analysis : Mesh influence on crack path

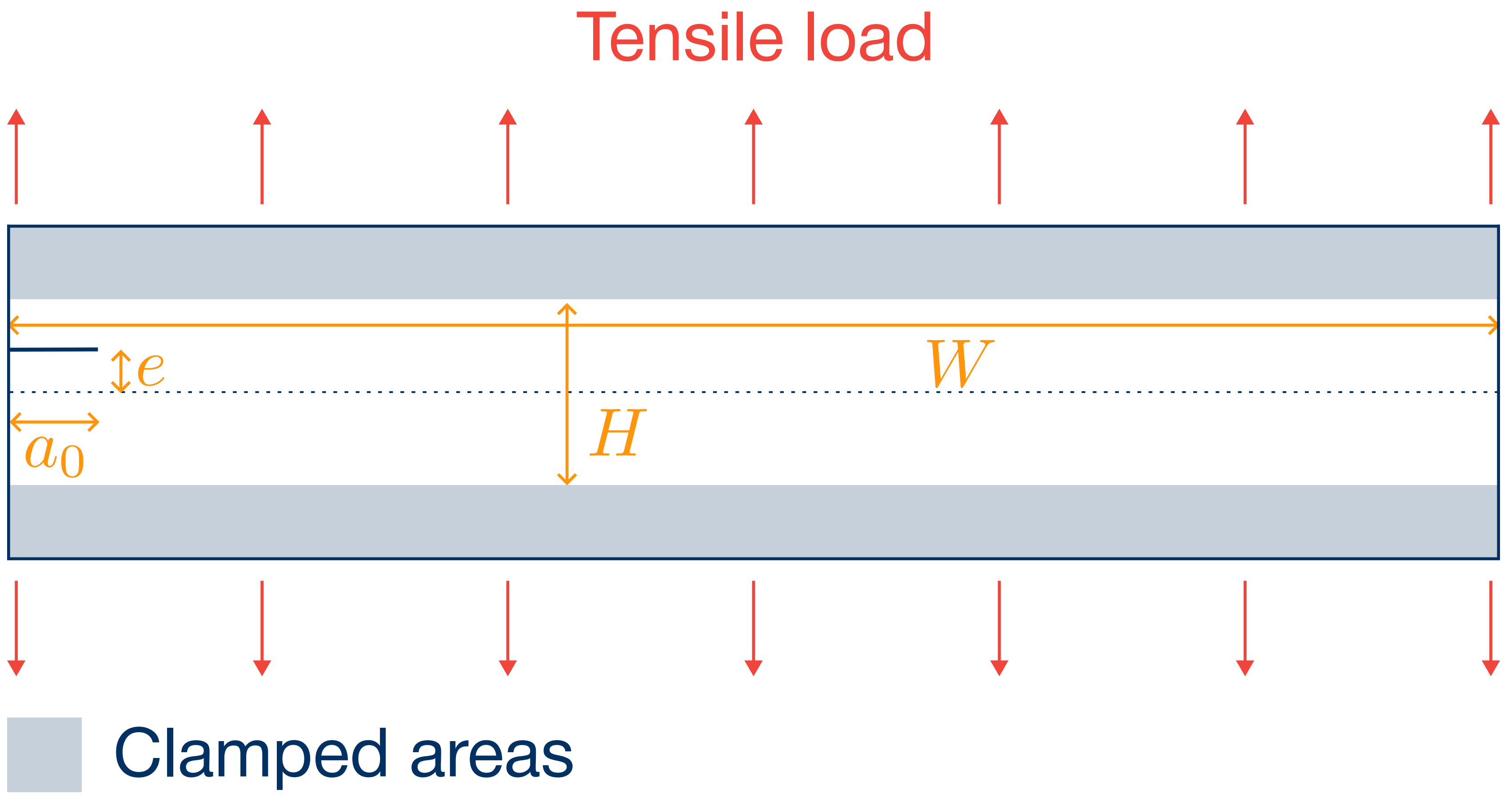
Objective

Preliminary analysis of the mesh influence on the crack path in variational phase-field models

Outline

- Presentation of the benchmark proposed by H. Henry
- Comparison of crack path on structured and unstructured meshes for:
  - Instable propagation case
  - Stable propagation case

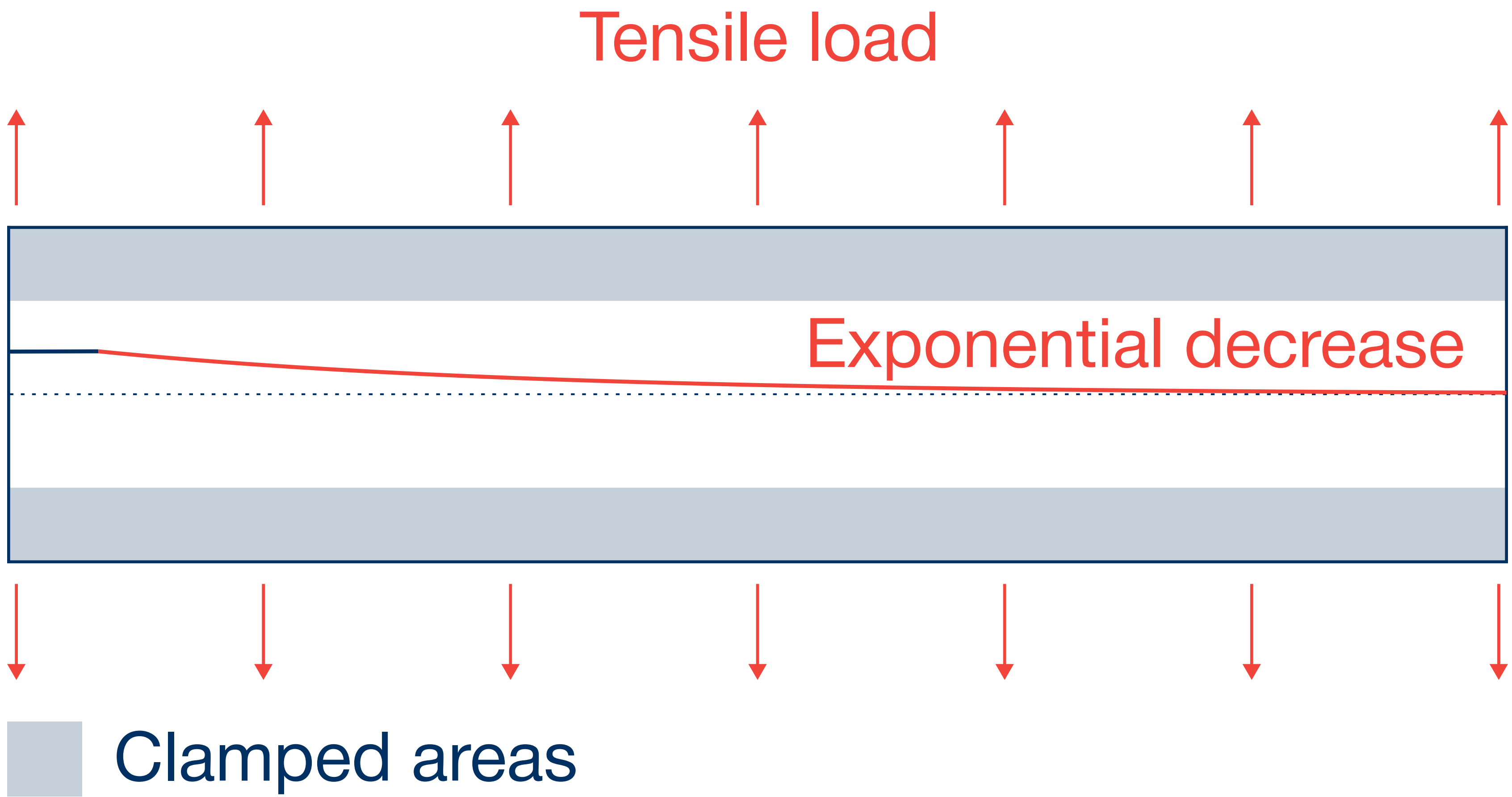
Benchmark: Eccentric Pure Shear (H. Henry)



$$H = 1 \qquad W = 8H \qquad a_0 = H/2 \qquad e = H/4$$

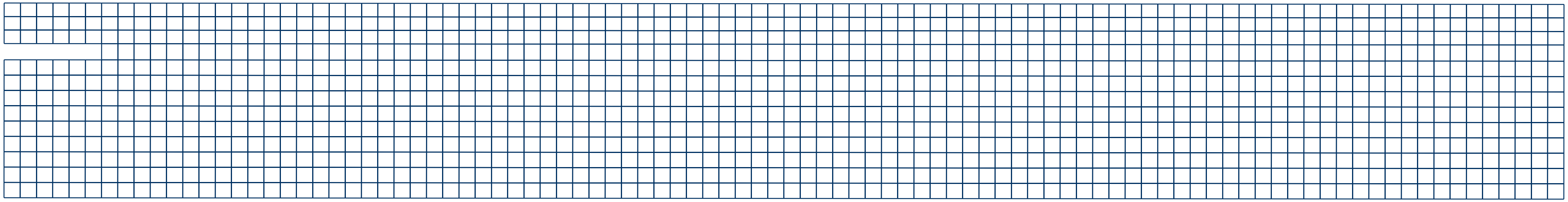
Other parameters are given at the end of the presentation (see appendix in Section 5.1)

Benchmark: Expected results

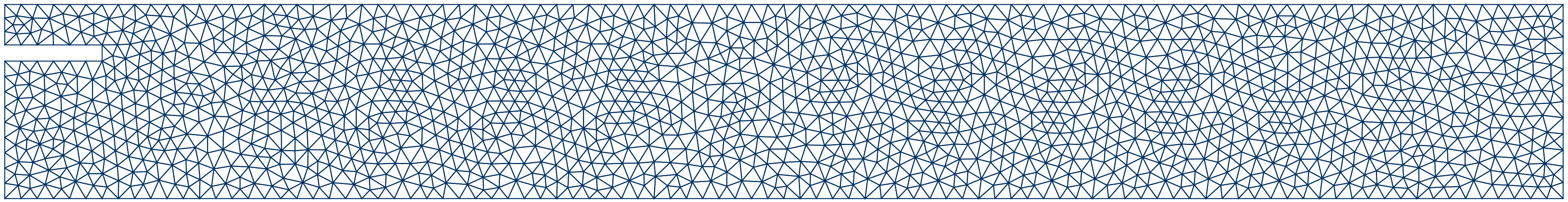


Two types of meshes

Structured mesh



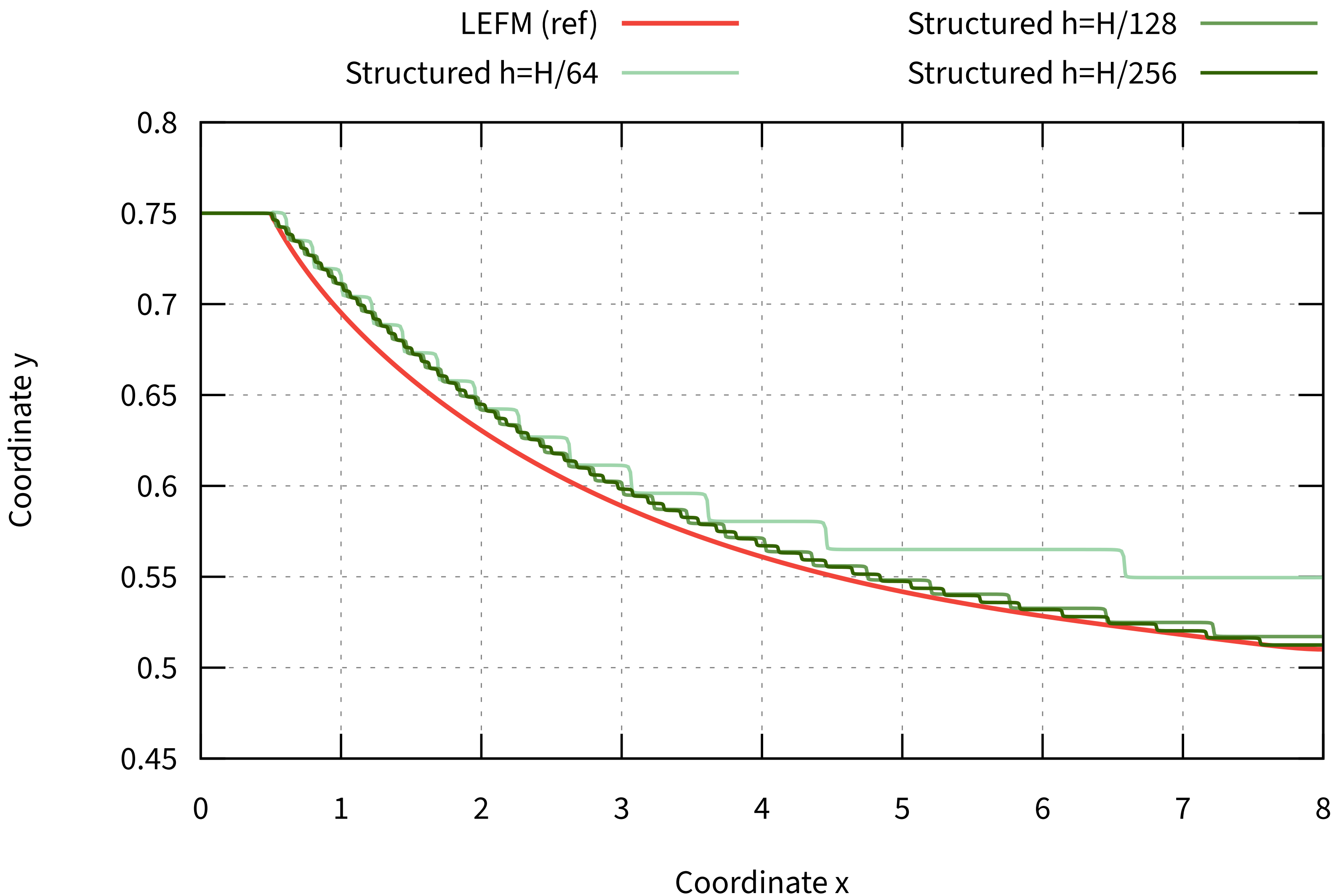
Unstructured mesh



Remarks

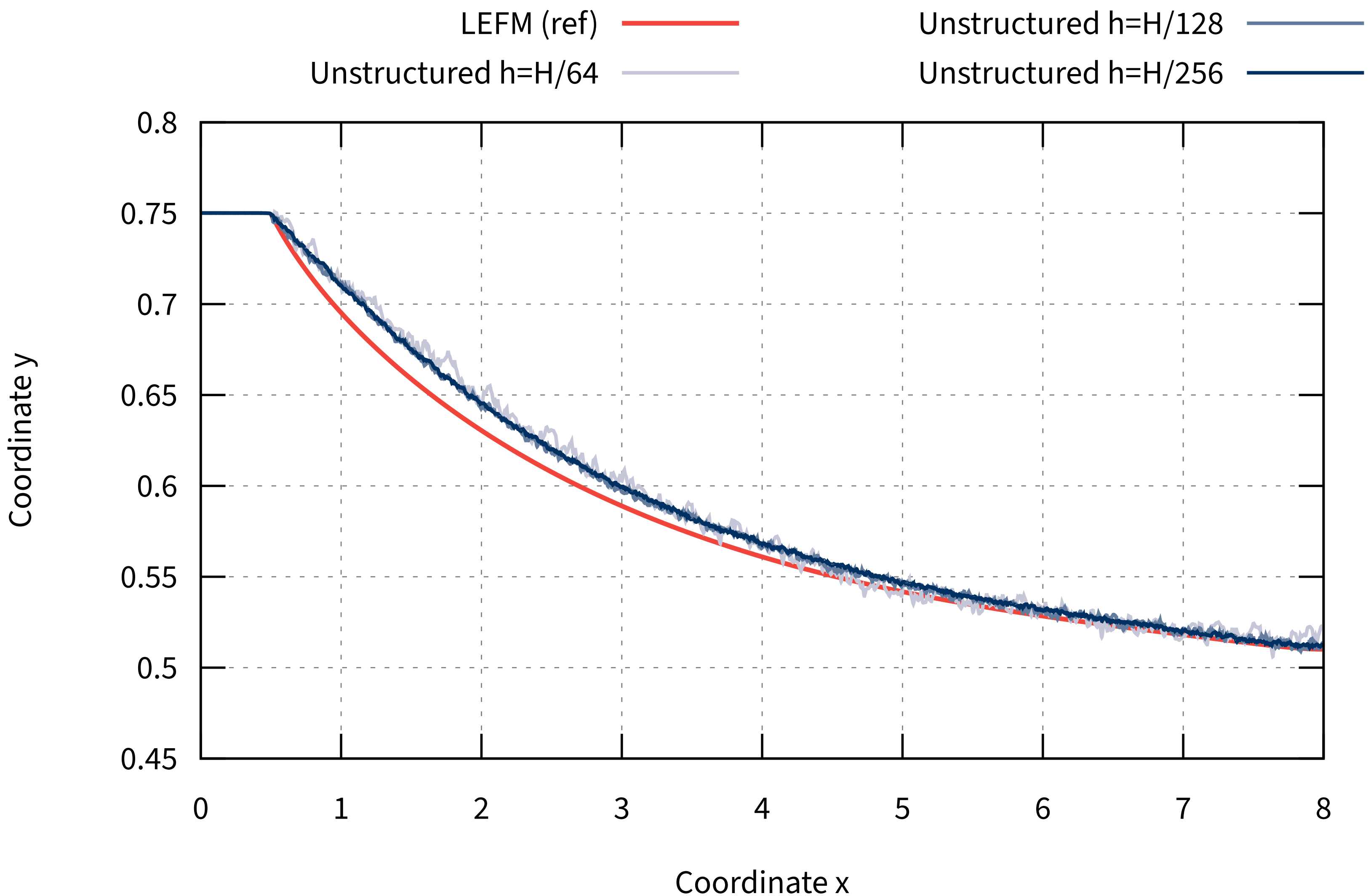
- Meshes have been coarsened for illustration purposes ($h = H/12$).
- Initial crack is one-element wide and $\alpha = 1$ on it (see preprint Loiseau & Lazarus (2025)).

Unstable propagation[1] – Structured mesh



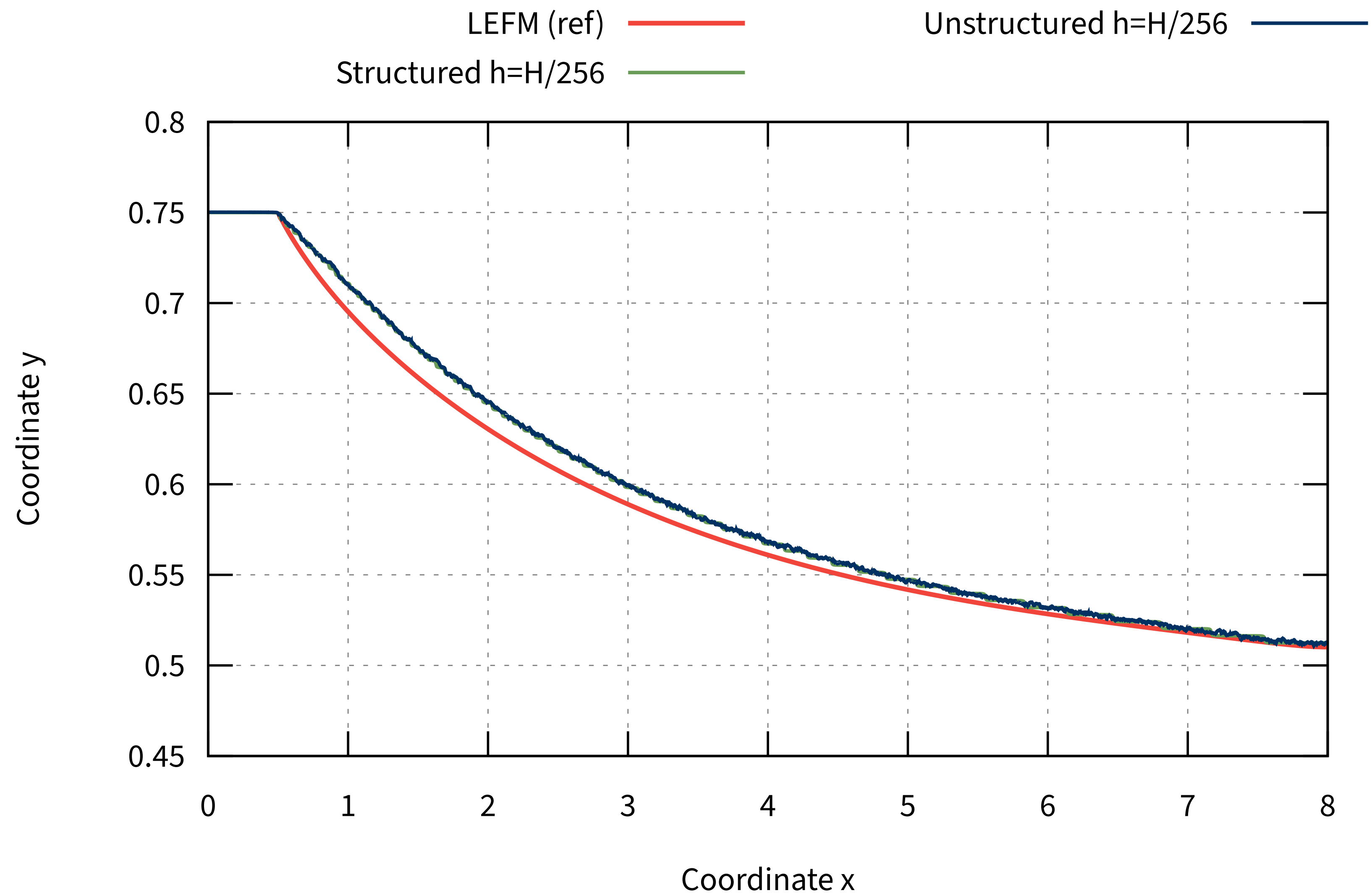
1. Whatever it means to solve a unstable problem in a quasi-static framework

Unstable propagation[1] – Unstructured mesh



1. Whatever it means to solve a unstable problem in a quasi-static framework

Unstable propagation[1] – Struct vs Unstruct



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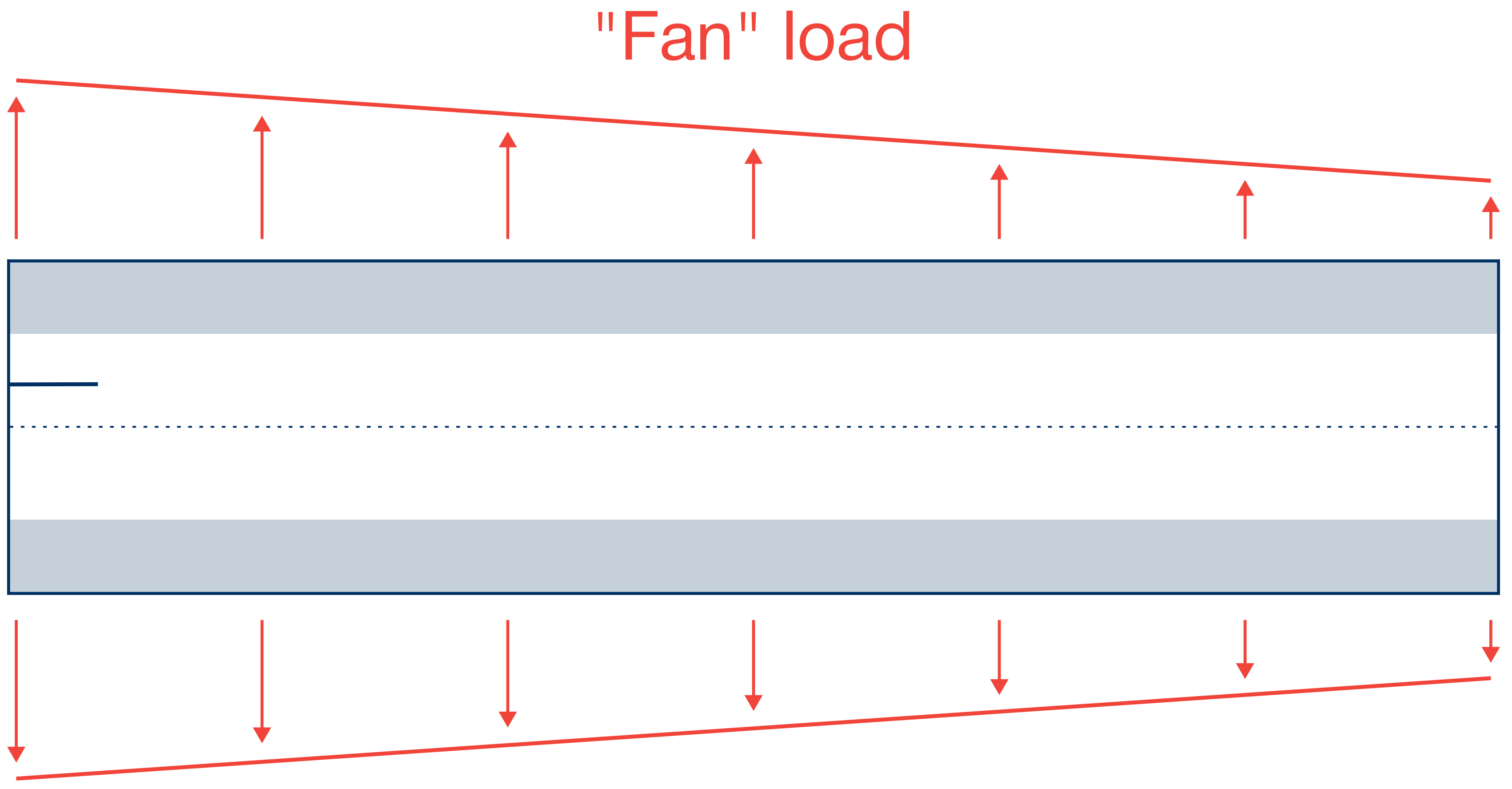
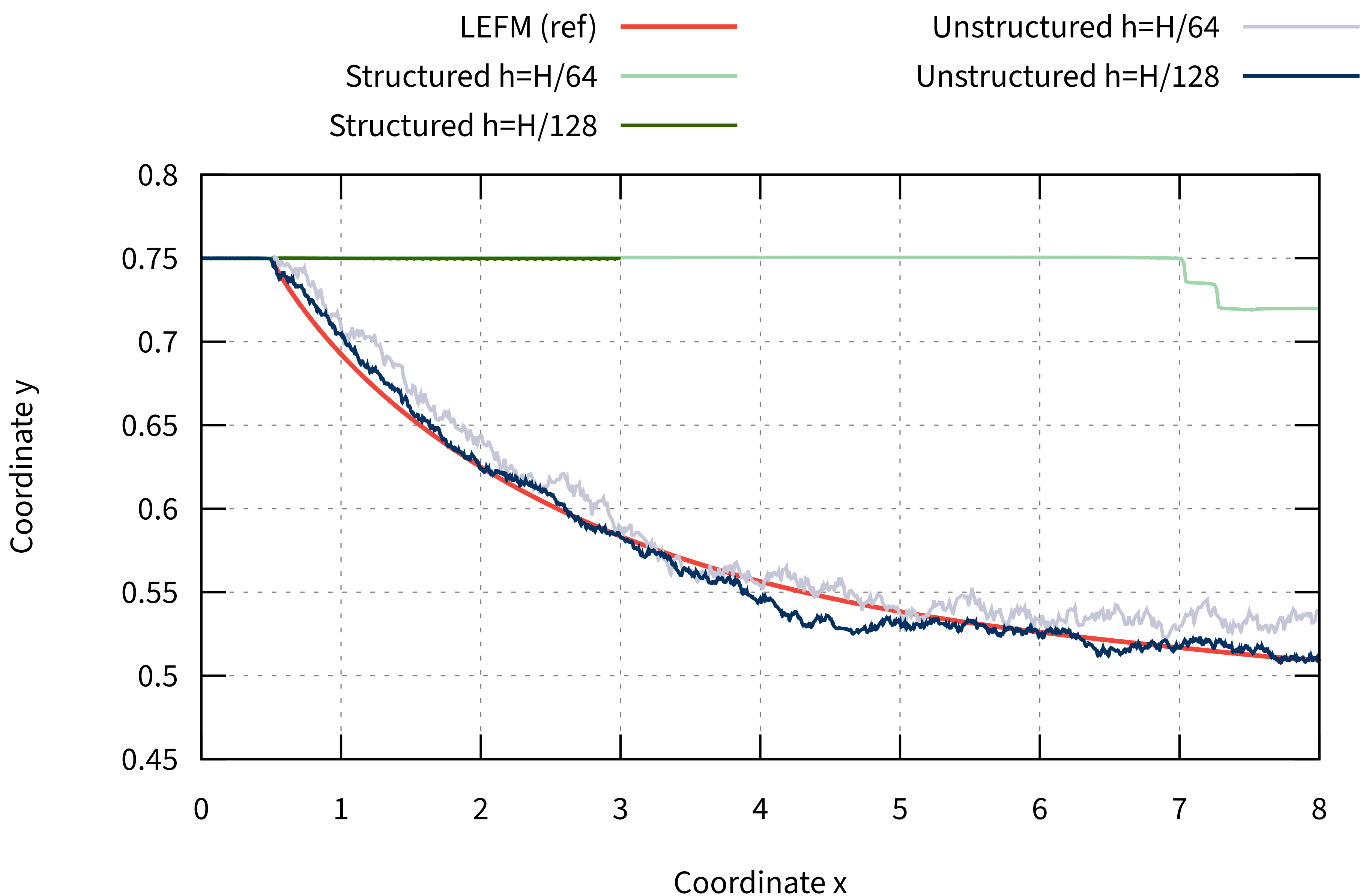
Stable crack propagation

Change of load



Stable crack propagation



Conclusion

Discussion of the results

Unstable crack propagation[1]

- Unstructured mesh : large bias which decreases with mesh refinement.
- Structured mesh : mesh only adds noise to the path.

Stable crack propagation

- Unstructured mesh : no convergence.
- Structured mesh : convergence but high noise.

Crack increments are more sensitive to the (local) discretization-induced anisotropy.

1. Once again ... whatever it means !

Summary

In this work, we:

- Propose a benchmark to assess the influence of the mesh on crack path.
- Encourage to extend mesh-sensitivity study to its structure (not only mesh size).
- Show that structured meshes severely bias the crack path (stable propagation).

Thank you for your attention !

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Presentation

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Appendices

Appendix : Parameters for the simulations

- 2D : plane stress.
- Phase-field model: AT1.
- Elastic parameters: $\lambda = \mu = 1Pa$.
- Fracture parameters: $G_c = 1$.
- Regularization length: $\ell = 0.0625 = H/16$.
- Mesh sizes: $h \in H/64, H/128, H/256$.
- Load $u_x = 0$ and $u_y(x,t) = \pm(t + \tan(\pi/180) * (H - x))$
  - Unstable : $\theta = 0$
  - Stable : $\theta = \pi/180$

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