Path-Following Methods for Quasi-Static Crack Propagation in Phase-Field Fracture Models

Flavien Loiseau   Véronique Lazarus

Quasi-static crack propagation simulations in brittle materials often suffer from numerical instabilities, such as snapback events, due to structural softening. In phase-field fracture models, these instabilities manifest as abrupt crack jumps, impeding the physical validity as the energy minimization is performed over a significant crack increment. Moreover, they often prevent the imposition of incremental force boundary conditions, as they may lead to a loss of force balance as soon as the crack starts to propagate.

This study explores the application of path-following methods to variational phase-field fracture simulation. It aims to identify a method that is robust, model- and problem-independent, and easy to implement in a classic staggered solver. To this aim, we evaluate existing path-following techniques alongside a novel proposition: Control by Maximum Strain Increment Outside the Crack (CMSIOC), which ensures stable crack propagation by limiting strain increments in the uncracked region. All studied methods introduce a single scalar control equation dependent solely on displacement fields, enabling seamless integration into classic staggered solvers without requiring significant modifications.

To assess the performance of these methods, we compare them across three numerical benchmark problems of increasing complexity. For reference, we employ a sharp crack model from Linear Elastic Fracture Mechanics (LEFM) based on Griffith's theory and the G-max criterion. Our findings demonstrate that CMSIOC reliably follows the equilibrium path, closely replicating LEFM benchmark solutions. Hence, it enables the capture of snapback instabilities without abrupt crack jumps, also allowing for force-controlled boundary conditions without loss of force balance. Another advantage is that it provides nearly uniform crack growth per increment, thereby balancing the computational cost across load steps.

This presentation will: (1) recall the concept of equilibrium paths in fracture mechanics, (2) introduce the CMSIOC, detailing its theoretical foundation and implementation, (3) present the results of the benchmark problems, and (4) conclude on this work with practical recommendations for integrating path-following methods into phase-field fracture simulations.