Formulation of anisotropic damage in quasi-brittle materials and structures based on discrete element simulations

Flavien Loiseau

Supervised by R. Desmorat, C. Oliver-Leblond

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Ph.D. Defense

LMPS Laboratoire de Mécanique Paris-Saclay

université PARIS-SACLAY

CentraleSupélec

école normale supérieure paris—saclay

cnrs

Usage of quasi-brittle materials

In civil engineering: Construction of structures

Energy

Buildings

Transportation

Why study quasi-brittle materials?

> Guarantee the integrity of structures during their life cycle
> Optimize our material usage
  - Cement ≈ 8% of $CO2$ emissions (Lehne & Preston, 2018)

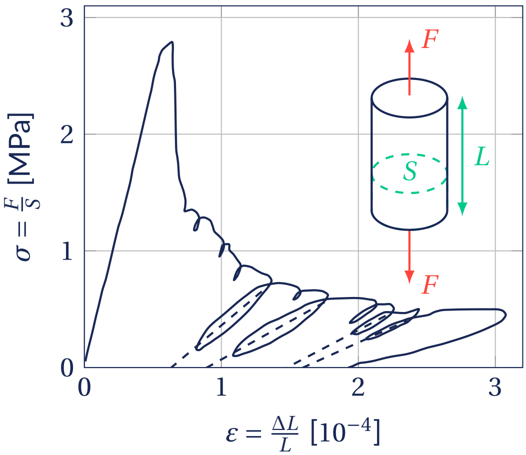
We need to

> understand how quasi-brittle materials degrade,
> model how the degradation impact on their behavior.

Illustrations from Wikimedia

Macroscopic observation of the degradation

(Terrien, 1980)



Observations

- Linear elastic phase
- Softening phase
- Linear unloading
- Permanent strain

Assumption

- Neglecting permanent strain

Source of the mechanical degradation

(Mac et al., 2021)

What happens ?

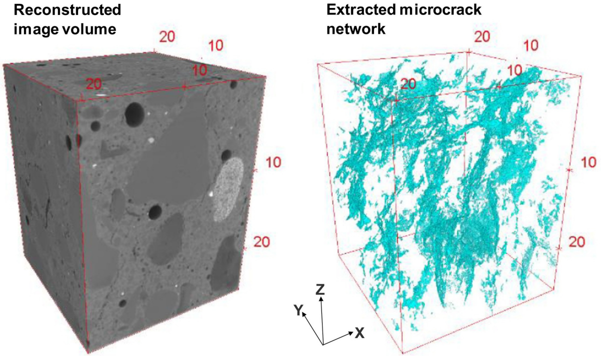
Mechanical loading

↓

Micro-cracks

↓

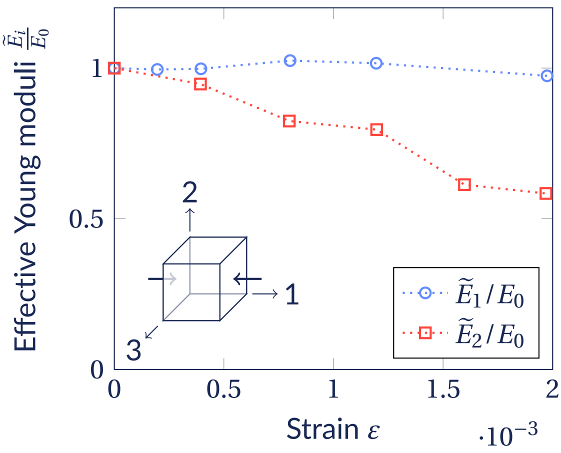
Degradation of mechanical properties



X-ray microtomography on concrete degraded due to shrinkage (sample diagonal 30 mm)

Another macroscopic observation: Damage-induced anistropy

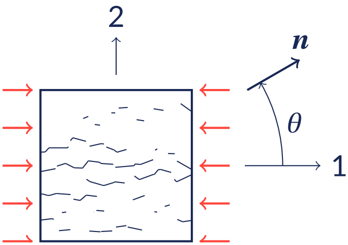
(Berthaud, 1991)



Observation

- Effective Young modulus $\widetilde{E}_i$ depends on the direction

Illustration in 2D

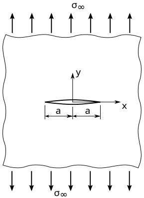


How to model the degradation?

How to model the degradation?

Fracture Mechanics

Linear Elastic FM (Griffith, 1921; Irwin, 1957)
Non-Linear FM (Rice, 1968)
Variational Approach (Francfort & Marigo, 1998)



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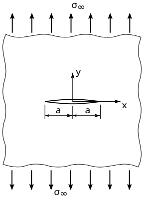

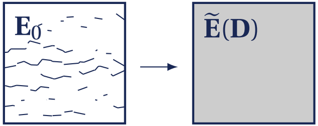

Continuum Damage Mechanics

Damage in creep (Kachanov, 1958; Rabotnov, 1969)
Effective stress (Lemaitre, 1971)
Non-local damage (Pijaudier-Cabot & Bažant, 1987)

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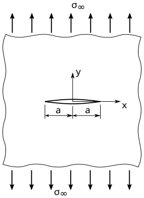

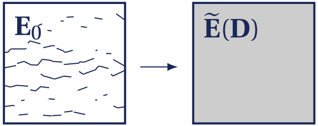

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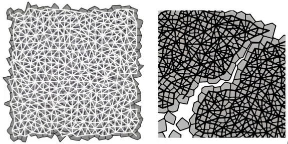
Discrete Models

Particle-based (DEM) (Cundall & Strack, 1979)
Lattice-based (Hrennikoff, 1941)
Hybrid (D'Addetta et al., 2002)



Formulation of a damage model

Damage model

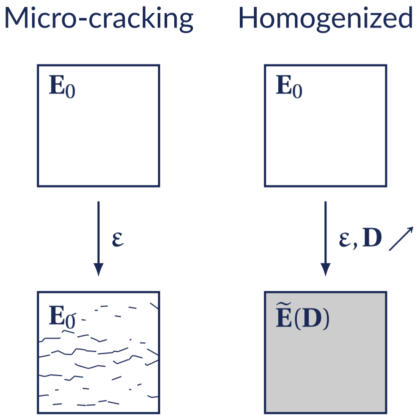
$\mathcal{V} = \{\varepsilon, \mathbf{D}, \ldots\}$ (State variables)

$\sigma = \widetilde{\mathbf{E}}(\mathbf{D}) : \varepsilon$ (State model)

$\dot{\mathbf{D}} = \ldots$ (Evolution law)

where

- $\mathbf{D}$ damage variable
- $\widetilde{\mathbf{E}}$ effective elasticity tensor



Constraints

- $\widetilde{\mathbf{E}}(\mathbf{D})$ is positive definite
- Positive dissipation

Reference books: (Lemaitre, 1992; Krajcinovic, 1996; Lemaitre & Desmorat, 2005; Murakami, 2012)

Remarks on Continuum Damage Mechanics for concrete

Phenomenological models

- Isotropic
  - (Mazars, 1984)
  - (Lubliner et al., 1989)
  - (Grassl & Jirásek, 2006)
  - (Richard et al., 2010)
- Anisotropic
  - (Murakami & Ohno, 1978)
  - (Halm & Dragon, 1996, 1998)
  - (Voyiadjis et al., 2008, 2022)
  - (Desmorat et al., 2007; Desmorat, 2016)

⚠ Coupling $\widetilde{\mathbb{E}}(\mathbf{D})$ between elasticity and damage is often simplified

Remarks on Continuum Damage Mechanics for concrete

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Micro-mechanics

> Homogenization of micro-cracked media
- (Vakulenko & Kachanov, 1971)
- (Kachanov, 1992)
- (Ponte Castañeda & Willis, 1995)
- (Cormery & Welemane, 2010)
- (Dormieux & Kondo, 2016)
- (Desmorat & Desmorat, 2016)

Limitations

Interactions between micro-cracks

Objectives

Thesis objective

Formulate an anisotropic damage model for quasi-brittle materials

Main focus

- Anisotropic damage
- Elasticity-damage coupling
  - Even at high level of damage

Secondary focus

- Damage evolution ?

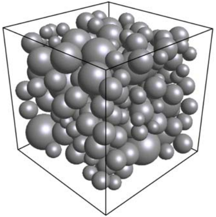
Assumptions

- Initial isotropy
- 2D case
- Micro-cracks closure neglected
  - No permanent strains
  - No stiffness recovery in compression

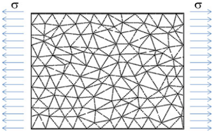
Virtual testing for study of concrete

Principle

Perform numerical experiment on a macro-element of the material using an accurate meso-scale model



(Wriggers & Moftah, 2006)
FEM simulations of concrete with explicit aggregates



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Disordered lattice simulations of heterogeneous quasi-brittle materials

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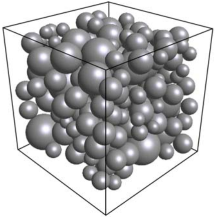

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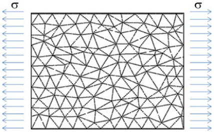

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Disordered lattice simulations of heterogeneous quasi-brittle materials

Advantages

- Efficient (simpler, faster)
- Versatile (different load cases)
- Access to full mechanical fields
- Reproducible

Limitations

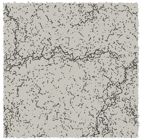
- Only as accurate as the model
- Unreal environment conditions

Methodology

Methodology

1. Virtual testing

Use of an accurate material model to perform numerical experiments and constitute the reference dataset



Degraded specimen

$$\rightarrow \widetilde{\mathbf{E}} = \begin{bmatrix} E_{1111} & E_{1122} & \sqrt{2}E_{1112} \\ E_{2211} & E_{2222} & \sqrt{2}E_{1222} \\ \sqrt{2}E_{1112} & \sqrt{2}E_{1222} & 2E_{2222} \end{bmatrix}$$

Effective elasticity tensor

Methodology

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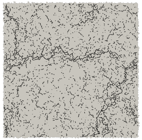

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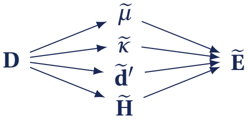
Effective elasticity tensor

2. State model

Determination of the coupling $\widetilde{\mathbf{E}}(\mathbf{D})$ between elasticity and damage from numerical experiments results

$$\boldsymbol{\sigma} = \widetilde{\mathbf{E}}(\mathbf{D}) : \boldsymbol{\varepsilon}$$



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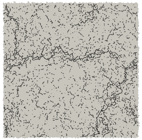

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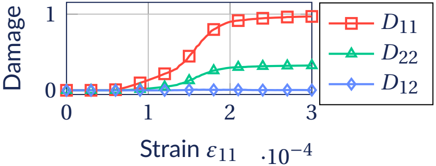
Determination of the coupling $\widetilde{\mathbf{E}}(\mathbf{D})$ between elasticity and damage from numerical experiments results

$$\boldsymbol{\sigma} = \widetilde{\mathbf{E}}(\mathbf{D}) : \boldsymbol{\varepsilon}$$

$$\mathbf{D} \rightarrow \widetilde{\mu}, \widetilde{\kappa}, \widetilde{\mathbf{d}}', \widetilde{\mathbf{H}} \rightarrow \widetilde{\mathbf{E}}$$

3. Evolution law

Analysis and determination of damage evolution $\dot{\mathbf{D}}$ during a mechanical loading



1. Virtual testing

Objective

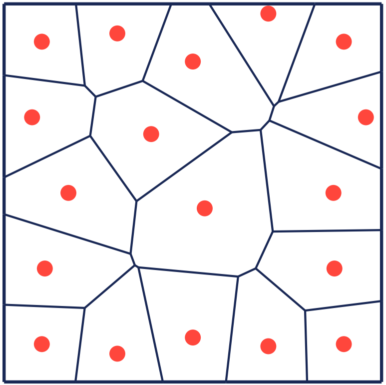
Generate a dataset of effective elasticity tensors evolution by virtual testing

Outline

- Describe the meso-scale (beam-particle) model
- Measure the evolution of an effective elasticity tensor
- Presentation of the generated reference dataset

Beam-particle model (Vassaux et al., 2016)

On the basis of Herrmann and Roux (1990), Delaplace et al. (1996), D'Addetta et al. (2002), and Delaplace (2008)



Components

> Rigid particles
  - random positions

Features

> Heterogeneous
> Explicit cracking
> Accurate failure (Oliver-Leblond, 2019)

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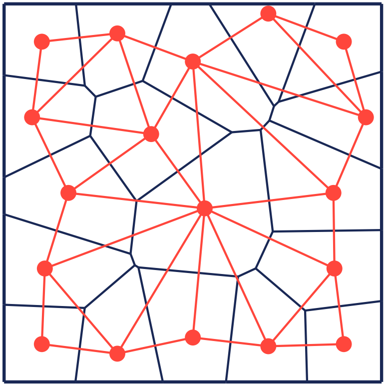

Components

- Rigid particles
  - random positions
- Euler-Bernoulli beams

Features

- Heterogeneous
- Explicit cracking
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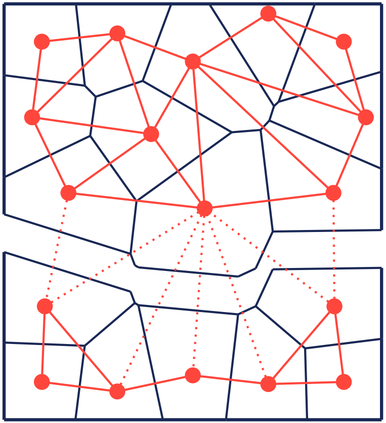


Components

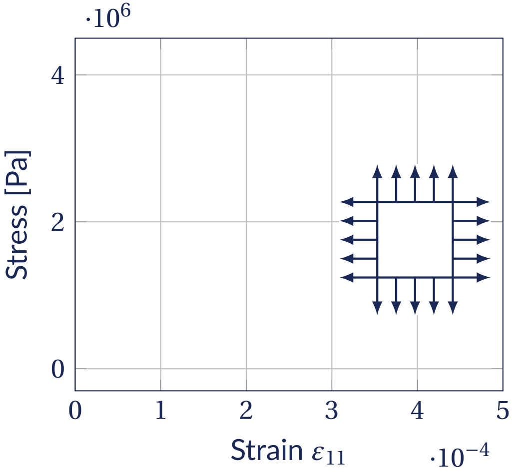
- Rigid particles
  - random positions
- Euler-Bernoulli beams
- Brittle beam failure
  - random thresholds
- Contact and friction (disabled)

Features

- Heterogeneous
- Explicit cracking
- Accurate failure (Oliver-Leblond, 2019)

Beam-particle model

Bitension loading - Periodic Boundary Conditions



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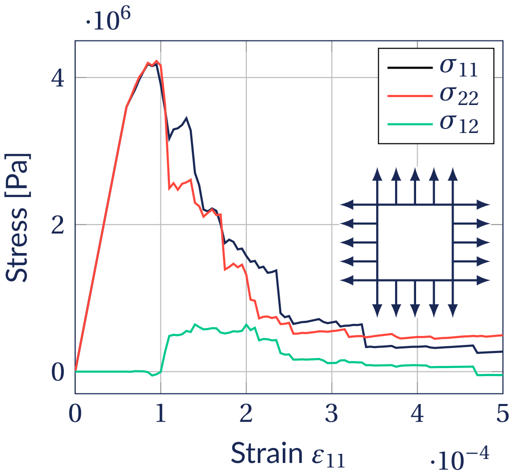

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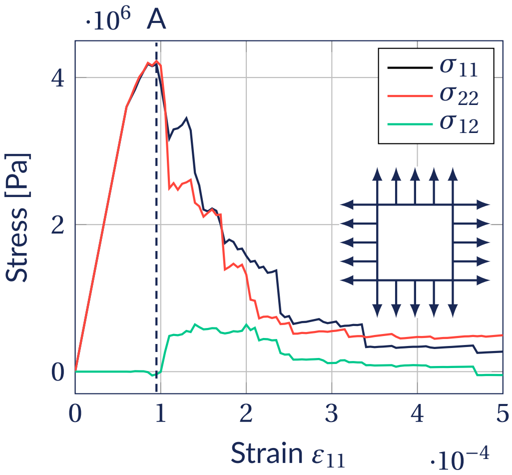

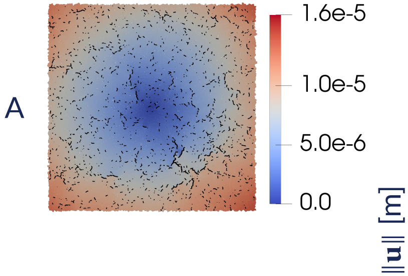

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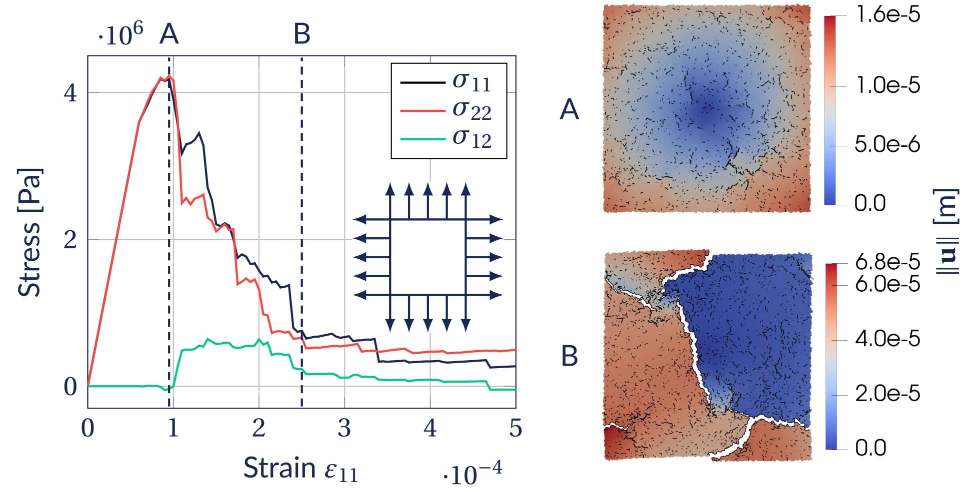
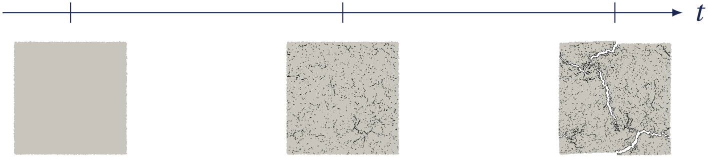


Procedure to measure effective elasticity tensors

Details on the measurement procedure in pp. 67–68 and 74

Procedure to measure effective elasticity tensors

Damaging loading



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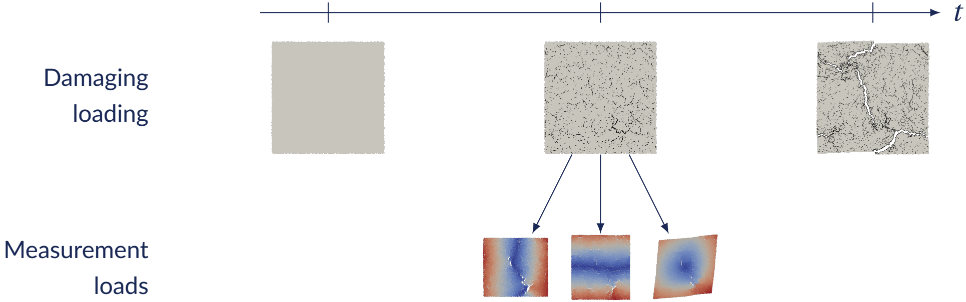

Details on the measurement procedure in pp. 67–68 and 74

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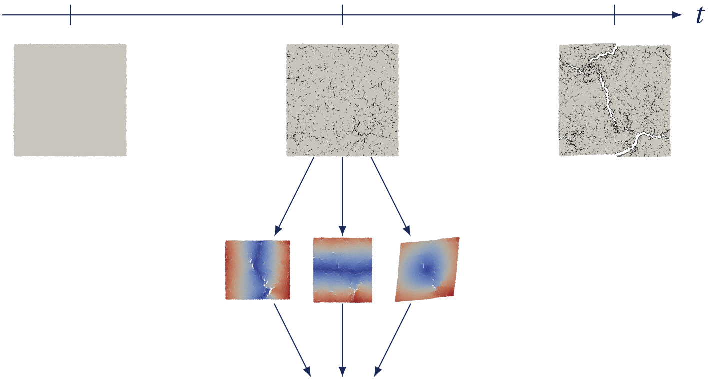
Damaging loading

Measurement loads



$\tilde{\mathbb{E}}$ [GPa]

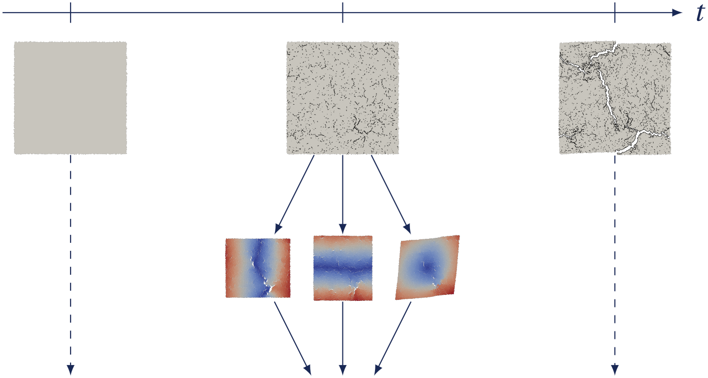
$$\begin{bmatrix} 33.5 & 5.62 & -0.39 \\ 5.62 & 36.0 & 0.34 \\ -0.39 & 0.34 & 28.0 \end{bmatrix}$$

Details on the measurement procedure in pp. 67–68 and 74

Procedure to measure effective elasticity tensors

Damaging loading

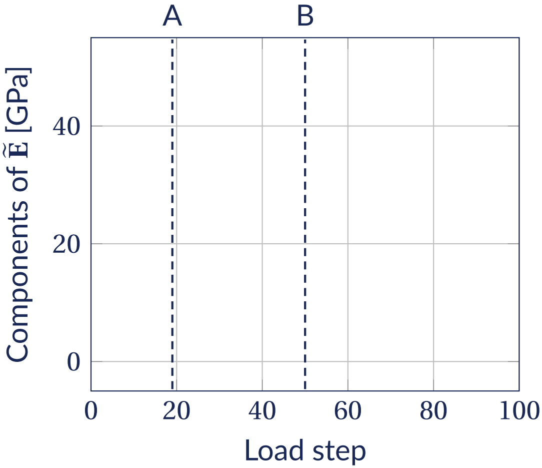
Measurement loads

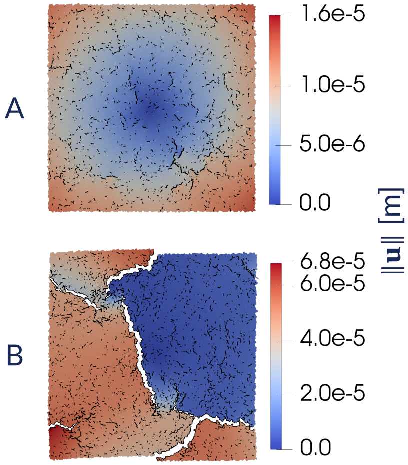


$$\tilde{\mathbb{E}}\ [\text{GPa}] \quad \begin{bmatrix} 49.8 & 10.2 & -0.01 \\ 10.2 & 49.8 & 0.01 \\ -0.01 & 0.01 & 38.4 \end{bmatrix} \begin{bmatrix} 33.5 & 5.62 & -0.39 \\ 5.62 & 36.0 & 0.34 \\ -0.39 & 0.34 & 28.0 \end{bmatrix} \begin{bmatrix} 3.82 & -1.27 & 0.14 \\ -1.27 & 3.30 & 0.82 \\ 0.14 & 0.82 & 6.84 \end{bmatrix}$$

Details on the measurement procedure in pp. 67–68 and 74

Application of the measurement procedure





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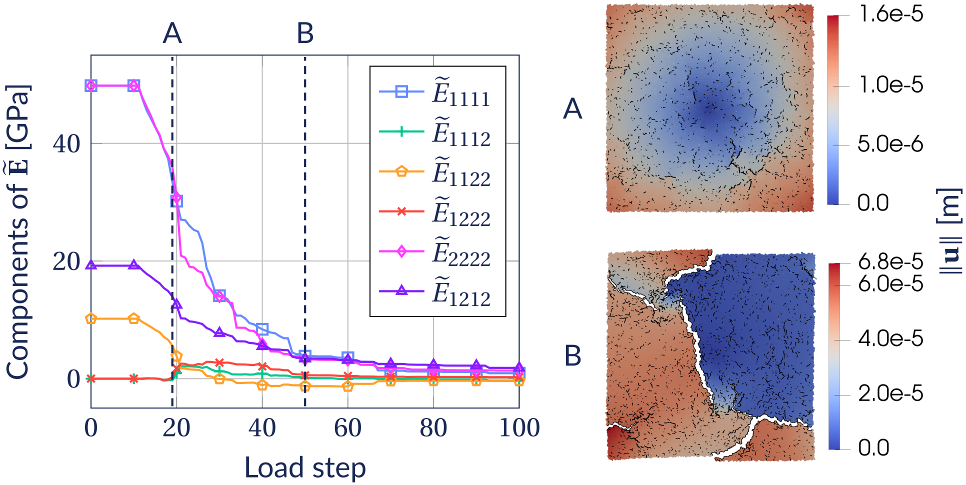


Generation of the reference dataset

Constitution

Repeat the procedure for

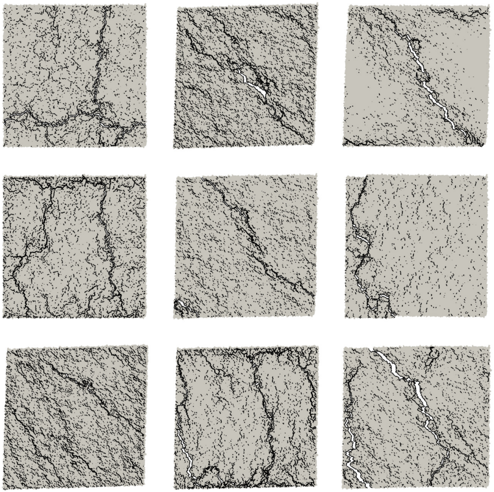
> 36 meso-structures,
- random particle position,
- random failure thresholds,

> 21 loadings,
- 100 load steps,

for a total of ≈ 76 000 tensors.

Dataset on Recherche Data Gouv

https://doi.org/10.57745/LYHM4W



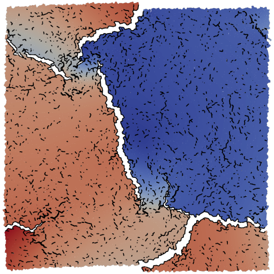
Limitations (positive definiteness, decrease of the effective elastic properties) detailed in Ch. 4, Sec. 3

Conclusion on virtual testing

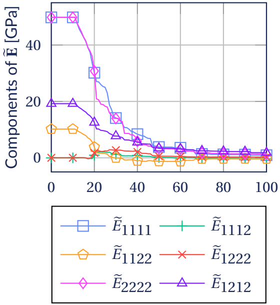
Objective reminder

Generate a dataset of effective elasticity tensor evolution by virtual testing

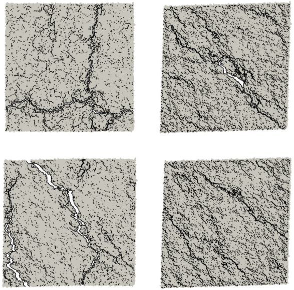
1. Beam-particle model



2. Measurement



3. Reference dataset



2. State model

Objective

Model the coupling between elasticity and anisotropic damage

Outline

1. Quantify micro-cracking by defining a damage variable $\mathbf{D}$
2. Model the impact of (anisotropic) damage on the effective elasticity
3. Assess the proposed model

Anisotropy: Distance to a symmetry class in 2D

(Vianello, 1997; Antonelli et al., 2022)

Question What tensorial order for the damage variable?

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Question What tensorial order for the damage variable?

Tool Relative distance to a symmetry stratum $\bar{\Sigma}$

$$\underbrace{\Delta_{\bar{\Sigma}}(\mathbf{E})}_{\in[0,1]} = \min_{\mathbf{E}^*\in\bar{\Sigma}} \frac{\|\mathbf{E}-\mathbf{E}^*\|}{\|\mathbf{E}\|}$$

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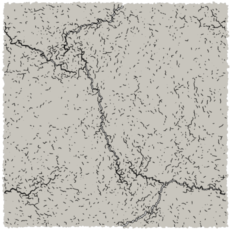
Illustration with the bitension loading



$\mathbf{E}$ [GPa]

$$\begin{bmatrix} 0.93 & -0.38 & -0.50 \\ -0.38 & 1.47 & 0.36 \\ -0.50 & 0.36 & 3.66 \end{bmatrix}$$

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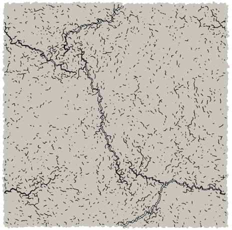

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Isotropy $\Delta_{\mathrm{Iso}} = 0.427$

$$\mathbf{E}_{\mathrm{Iso}} = \begin{bmatrix} 1.68 & -0.91 & 0.00 \\ -0.91 & 1.68 & 0.00 \\ 0.00 & 0.00 & 2.59 \end{bmatrix}$$

Anisotropy: Distance to a symmetry class in 2D

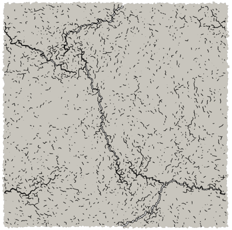
(Vianello, 1997; Antonelli et al., 2022)

Question What tensorial order for the damage variable?

Tool Relative distance to a symmetry stratum $\bar{\Sigma}$

$$\underbrace{\Delta_{\bar{\Sigma}}(\mathbf{E})}_{\in[0,1]} = \min_{\mathbf{E}^*\in\bar{\Sigma}} \frac{\|\mathbf{E}-\mathbf{E}^*\|}{\|\mathbf{E}\|}$$

Illustration with the bitension loading



$\mathbf{E}$ [GPa]

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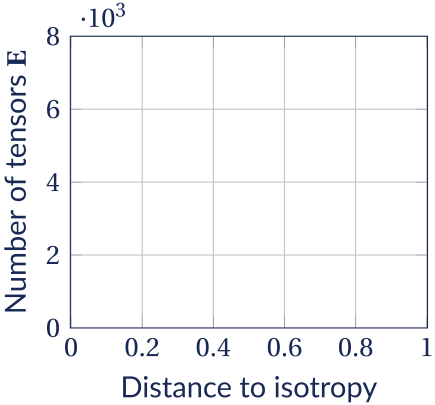
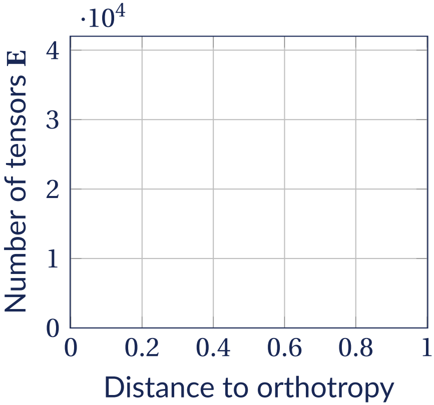
Orthotropy $\Delta_{\text{Ort}} = 0.013$

$$\mathbf{E}_{\text{Ort}} = \begin{bmatrix} 0.92 & -0.38 & -0.48 \\ -0.38 & 1.38 & 0.39 \\ -0.48 & 0.39 & 3.66 \end{bmatrix}$$

Anisotropy of the effective elasticity tensors





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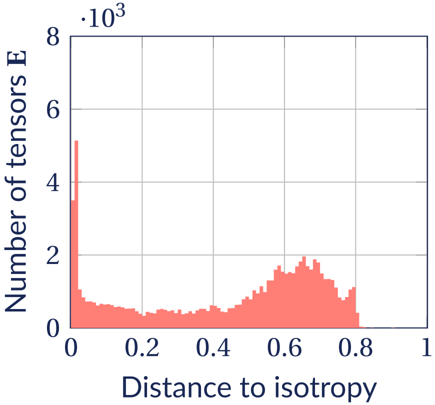

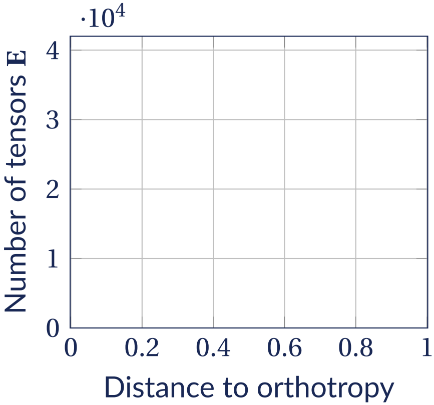

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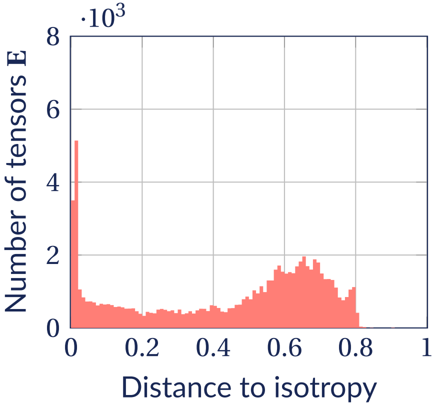
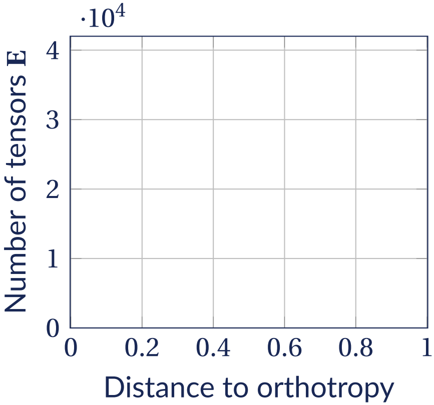
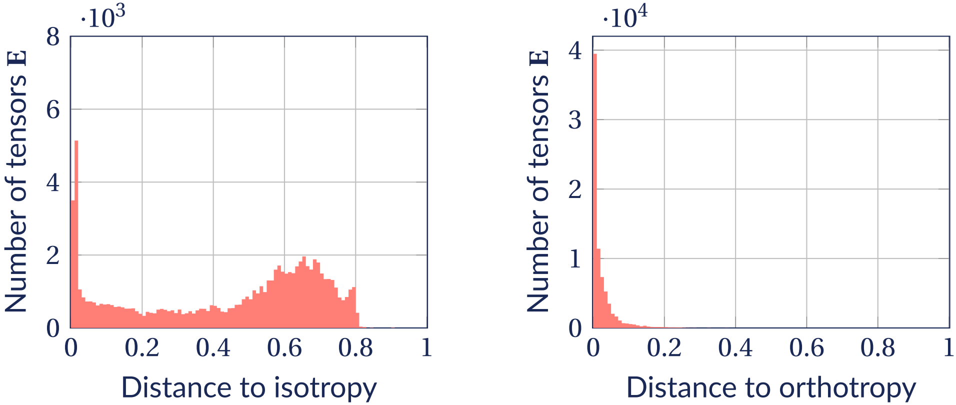




✗ Scalar damage

Anisotropy of the effective elasticity tensors



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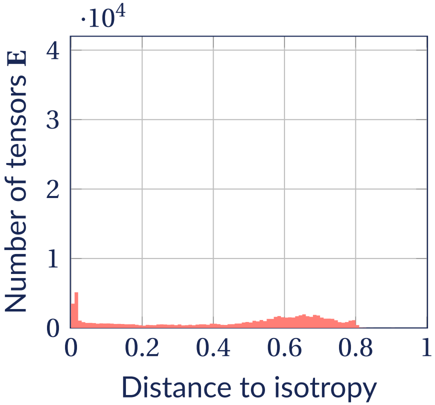

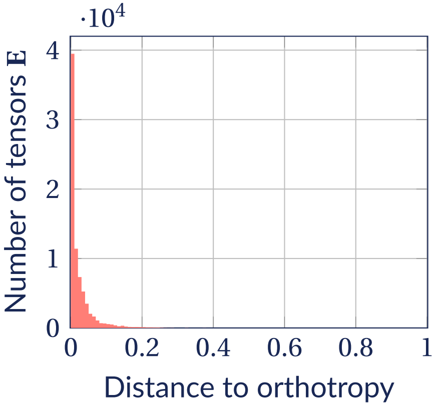

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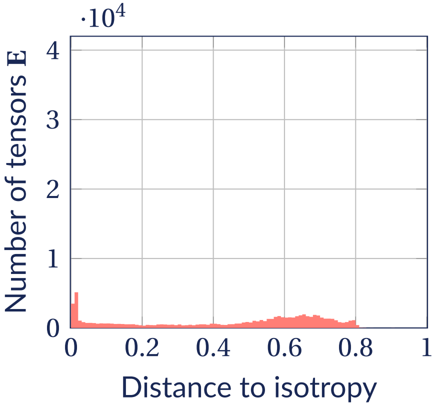

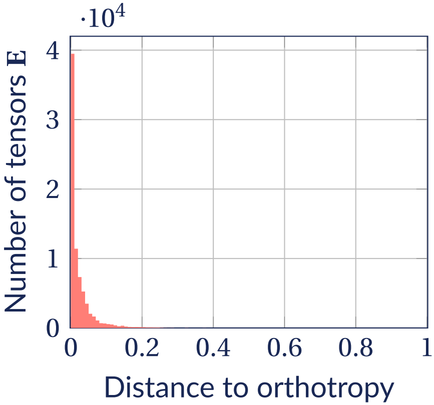

✗ Scalar damage

✓ At least 2nd order tensor

Harmonic decomposition in 2D

Applications to elasticity tensor: 3D Backus (1970), 2D Blinowski et al. (1996)

Elasticity tensor $\mathbf{E}$ in $\mathbb{E}\mathrm{la}(\mathbb{R}^2)$

$\mathbb{H}^n(\mathbb{R}^2)$: space of nth-order harmonic tensors (totally symmetric and traceless).

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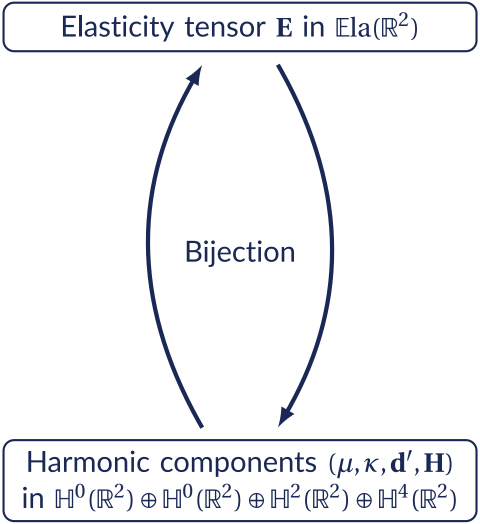

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Applications to elasticity tensor: 3D Backus (1970), 2D Blinowski et al. (1996)

Elasticity tensor $\mathbf{E}$ in $\mathbb{E}\text{la}(\mathbb{R}^2)$

$$\mathbf{E} = \mathbf{Iso} + \mathbf{Dil} + \mathbf{H}$$

$$\mathbf{Iso} = 2\mu\mathbf{J} + \kappa\mathbf{1}\otimes\mathbf{1}$$

$$\mathbf{Dil} = \frac{1}{2}\left(\mathbf{d}'\otimes\mathbf{1} + \mathbf{1}\otimes\mathbf{d}'\right)$$

Bijection

Harmonic components $(\mu, \kappa, \mathbf{d}', \mathbf{H})$ in $\mathbb{H}^0(\mathbb{R}^2)\oplus\mathbb{H}^0(\mathbb{R}^2)\oplus\mathbb{H}^2(\mathbb{R}^2)\oplus\mathbb{H}^4(\mathbb{R}^2)$

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$$\kappa = \tfrac{1}{4}\,\mathrm{tr}\,\mathbf{d}$$
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$$\mathbf{H} = \mathbf{E} - \mathbf{Iso} - \mathbf{Dil}$$

$$\mathbf{d} = \mathrm{tr}_{12}\,\mathbf{E}$$
$$\mathbf{v} = \mathrm{tr}_{13}\,\mathbf{E}$$

Harmonic components $(\mu, \kappa, \mathbf{d}', \mathbf{H})$
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Principle of the model and definition of damage

(Oliver-Leblond et al., 2021)

Knowing isotropic $\mathbf{E}_0 \leftrightarrow (\mu_0, \kappa_0, \mathbf{0}, \mathbf{0})$ and $\mathbf{D}$, we want to model

$$\widetilde{\mathbf{E}}(\mathbf{D}) = 2\widetilde{\mu}(\mathbf{D})\mathbf{J} + \widetilde{\kappa}(\mathbf{D})\mathbf{1}\otimes\mathbf{1} + \frac{1}{2}\left(\widetilde{\mathbf{d}}'(\mathbf{D})\otimes\mathbf{1} + \mathbf{1}\otimes\widetilde{\mathbf{d}}'(\mathbf{D})\right) + \widetilde{\mathbf{H}}(\mathbf{D})$$

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How to define damage? Using the harmonic decomposition

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Damage variable

$$\mathbf{D} = \underbrace{(\mathbf{d}_0 - \mathbf{d})\cdot\mathbf{d}_0^{-1}}_{\substack{\text{normalize } \mathbf{d} \\ \mathbf{d}_0 = 2\kappa_0\mathbf{1}}} = \mathbf{1} - \frac{1}{2\kappa_0}\mathbf{d} \iff \mathbf{d} = 2\kappa_0(\mathbf{1} - \mathbf{D})$$

Bulk modulus and dilatation tensor as functions of damage

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$$\Downarrow$$

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$$\widetilde{\mathbf{d}}'(\mathbf{D}) = -2\kappa_0\mathbf{D}'$$

Summary of the partial state model

Knowing isotropic $\mathbf{E}_0 \leftrightarrow (\mu_0, \kappa_0, \mathbf{0}, \mathbf{0})$ and $\mathbf{D}$,

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where decomposition $\mathbf{E} \mapsto (\mu, \kappa, \mathbf{d}', \mathbf{H})$ and damage definition $\mathbf{d} \mapsto \mathbf{D}$ give

$$\mu(\mathbf{E}) = \frac{1}{8}(2\,\mathrm{tr}\,\mathbf{v} - \mathrm{tr}\,\mathbf{d}) \qquad \widetilde{\mathbf{d}}'(\mathbf{D}) = -2\kappa_0 \mathbf{D}'$$

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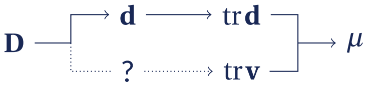
Questions How to model ○ shear modulus $\widetilde{\mu}(\mathbf{D})$? ○ harmonic part $\widetilde{\mathbf{H}}(\mathbf{D})$?

Modelling $\mu = \frac{1}{8}\left(2\,\mathrm{tr}\,\mathbf{v} - \mathrm{tr}\,\mathbf{d}\right)$



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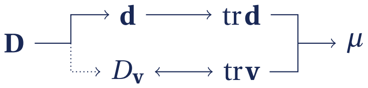

$D_{\mathbf{v}}$ such that $\mathrm{tr}\,\mathbf{v} = \mathrm{tr}\,\mathbf{v}_0(1 - D_{\mathbf{v}})$

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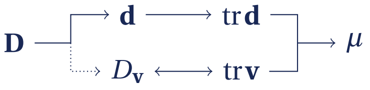

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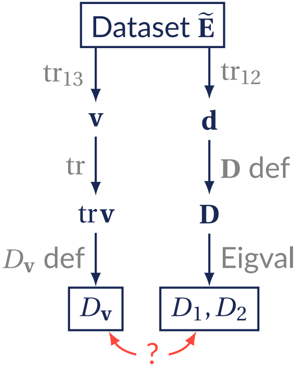

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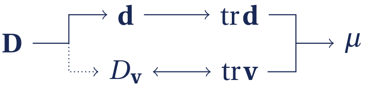

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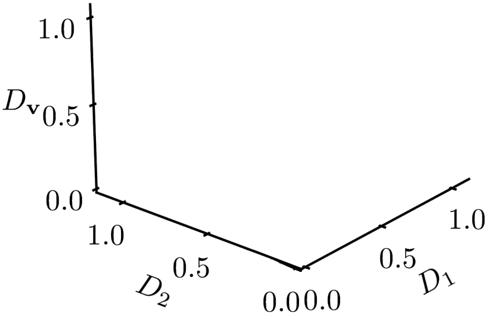

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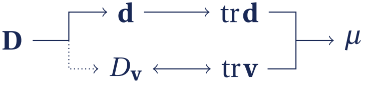

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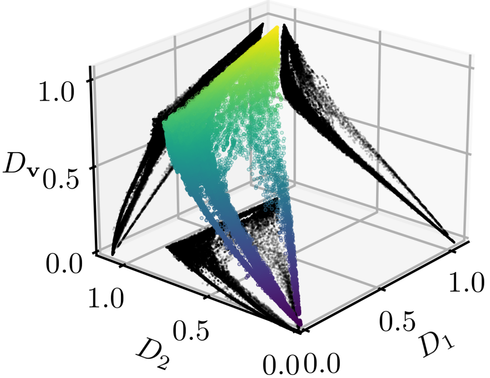

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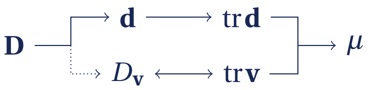

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Assumptions

$\tilde{\mu}(\mathbf{D} = \mathbf{0}) = \mu_0$ (Initial)
$\tilde{\mu}(\mathbf{D} = \mathbf{1}) = 0$ (Full damage)
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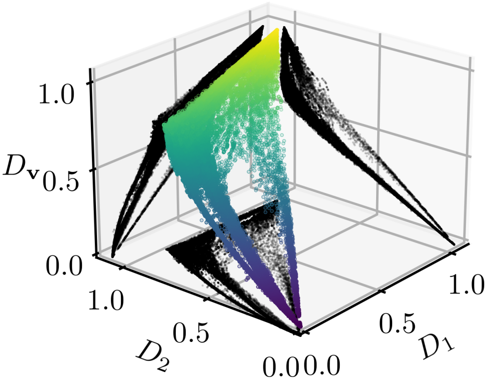

* Early damage $\Longrightarrow$ Non-interacting cracks $\Longrightarrow$ Tot. sym. stiffness loss (Kachanov, 1992)

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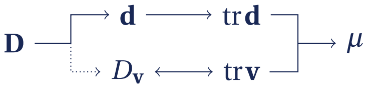

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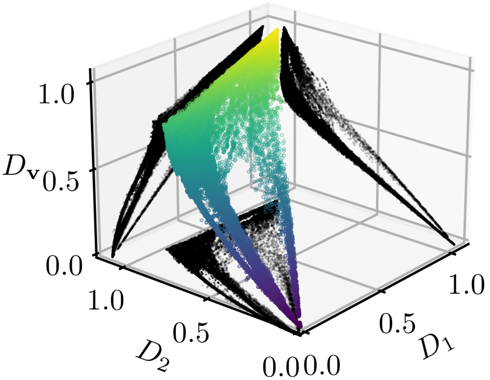
We model $D_\mathbf{v}$ as linear combination of damage invariants

$$I_n(\mathbf{D}) = \mathrm{tr}(\mathbf{D}^n) = D_1^n + D_2^n$$



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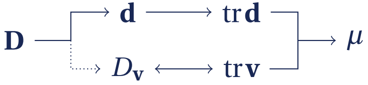

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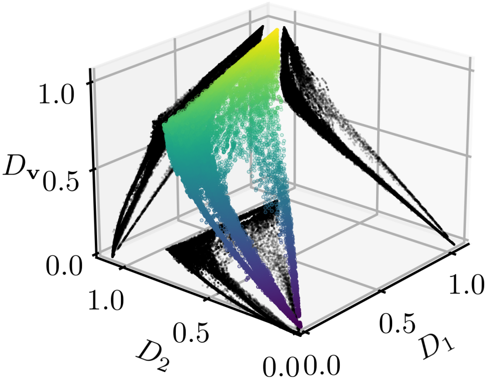

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with 2 invariants: $D_\mathbf{v}^\mathrm{m} = c_1 I_1(\mathbf{D}) + c_2 I_2(\mathbf{D})$

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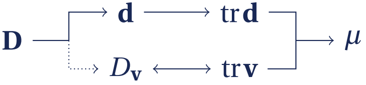

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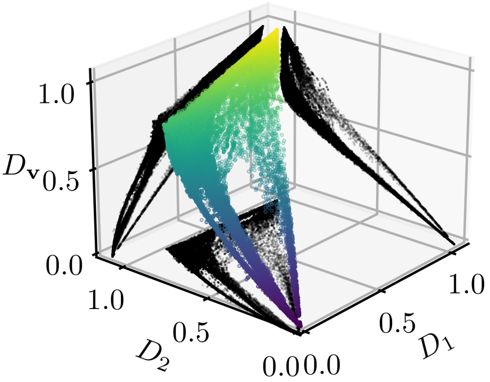
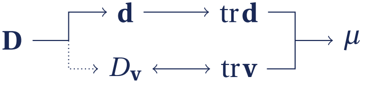


with 2 invariants: $$D_{\mathbf{v}}^{\mathrm{m}} = c_1 I_1(\mathbf{D}) + c_2 I_2(\mathbf{D}) \implies c_1 = \frac{\kappa_0}{2\mu_0 + \kappa_0},\; c_2 = \frac{1}{2} - c_1.$$

* Early damage $\implies$ Non-interacting cracks $\implies$ Tot. sym. stiffness loss (Kachanov, 1992)

Modelling $\mu = \frac{1}{8}(2\,\mathrm{tr}\,\mathbf{v} - \mathrm{tr}\,\mathbf{d})$



$D_{\mathbf{v}}$ such that $\mathrm{tr}\,\mathbf{v} = \mathrm{tr}\,\mathbf{v}_0(1 - D_{\mathbf{v}})$

Assumptions

$\tilde{\mu}(\mathbf{D} = \mathbf{0}) = \mu_0$	(Initial)
$\tilde{\mu}(\mathbf{D} = \mathbf{1}) = 0$	(Full damage)
$\mathrm{tr}\,\mathbf{d} = \mathrm{tr}\,\mathbf{v}$	(Early*, $\mathbf{D} \approx \mathbf{0}$)

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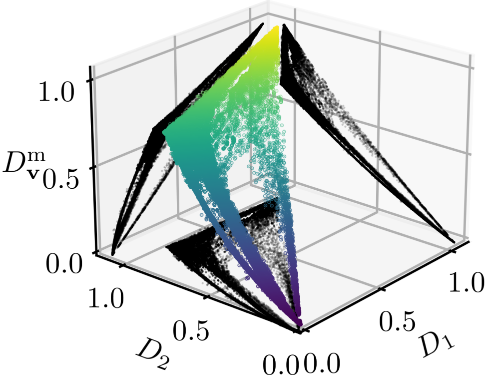

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Summary of the (still) partial state model

Knowing isotropic $\mathbf{E}_0 \leftrightarrow (\mu_0, \kappa_0, \mathbf{0}, \mathbf{0})$ and $\mathbf{D}$,

$$\widetilde{\mathbf{E}}(\mathbf{D}) = 2\widetilde{\mu}(\mathbf{D})\mathbf{J} + \widetilde{\kappa}(\mathbf{D})\mathbf{1}_2 \otimes \mathbf{1}_2 + \frac{1}{2}\left(\widetilde{\mathbf{d}}'(\mathbf{D}) \otimes \mathbf{1}_2 + \mathbf{1}_2 \otimes \widetilde{\mathbf{d}}'(\mathbf{D})\right) + \widetilde{\mathbf{H}}(\mathbf{D})$$

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$$\widetilde{\mu}(\mathbf{D}) = \mu_0 - \frac{1}{4}\kappa_0(\operatorname{tr}\mathbf{D}) + \frac{1}{4}(\kappa_0 - 2\mu_0)(\mathbf{D}:\mathbf{D}) \qquad \widetilde{\mathbf{d}}'(\mathbf{D}) = -2\kappa_0\mathbf{D}'$$

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Questions How to model ✔ shear modulus $\widetilde{\mu}(\mathbf{D})$? ○ harmonic part $\widetilde{\mathbf{H}}(\mathbf{D})$?

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Modelling the harmonic part $\mathbf{H}$

Parametrization based on Vannucci (2005) and Desmorat and Desmorat (2015)

How to parametrize the harmonic part?

$$\text{Orthotropy} \implies \mathbf{H} = \|\mathbf{H}\| \left(\pm \frac{\mathbf{d}' * \mathbf{d}'}{\|\mathbf{d}' * \mathbf{d}'\|} \right)$$

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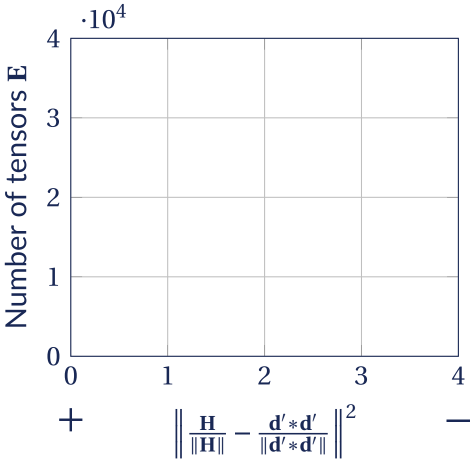

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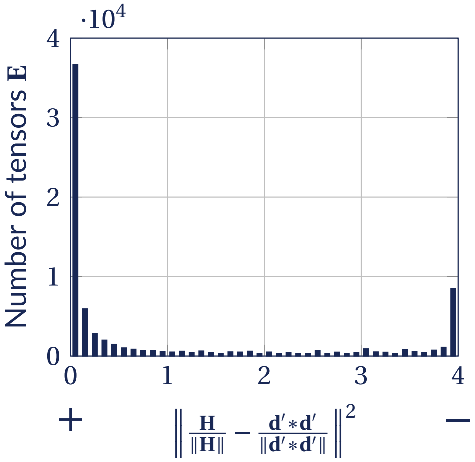

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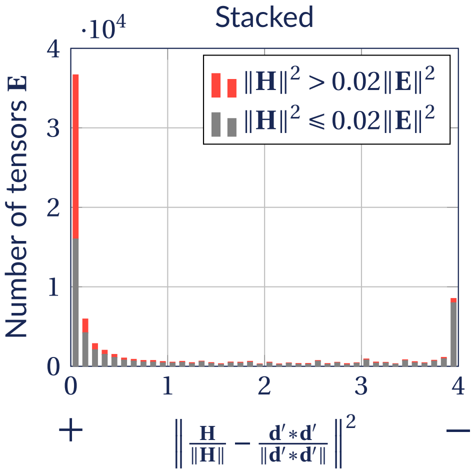

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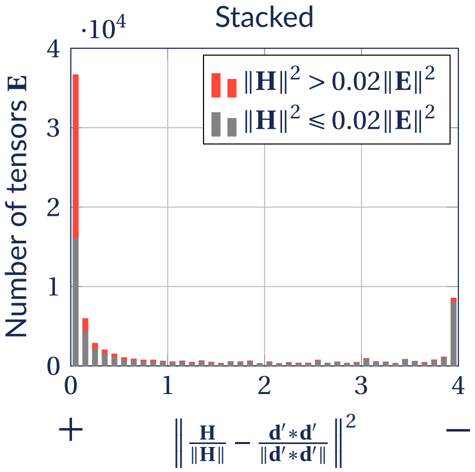
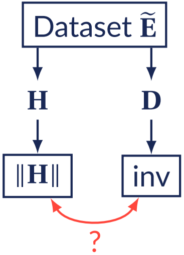


Modelling the harmonic part $\mathbf{H}$

Norm modelling $H^{\mathrm{m}} : \mathbf{D} \mapsto H^{\mathrm{m}}(\mathbf{D}) \approx \|\mathbf{H}\|$?



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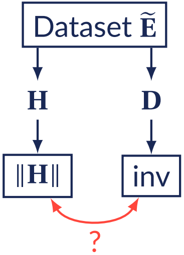

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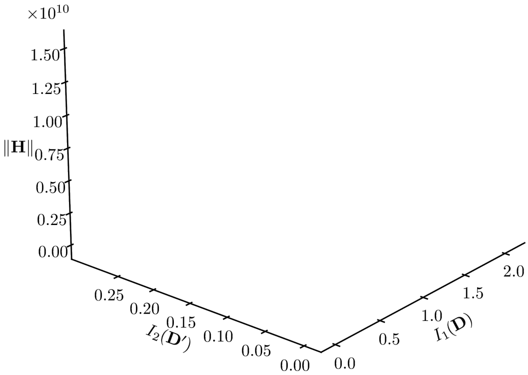

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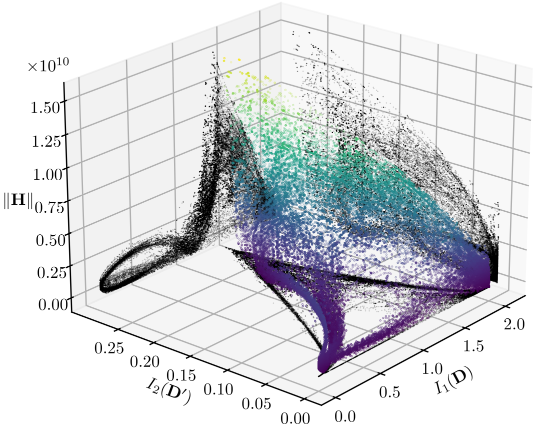

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$$H^{\mathrm{m}}(\mathbf{D} = \mathbf{0}) = 0 \qquad \text{(Initial isotropy)}$$

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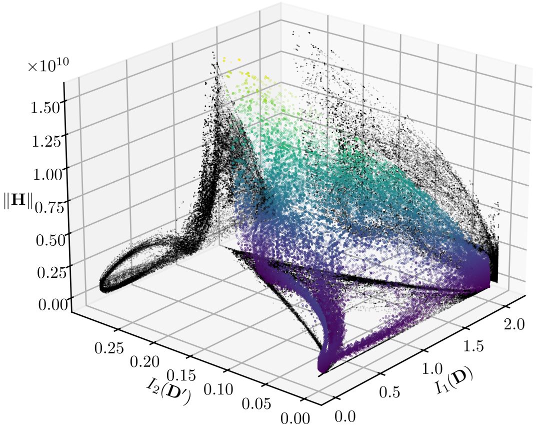

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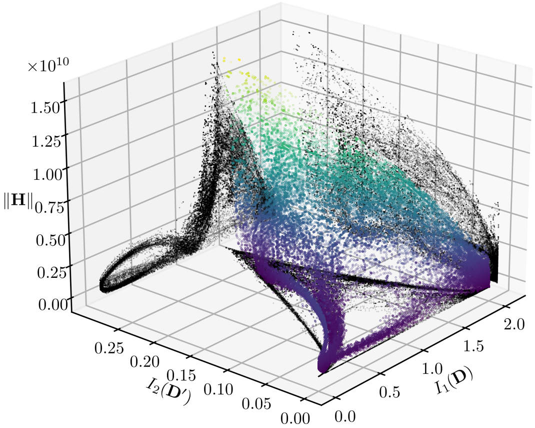
$$H^{\mathrm{m}}(\mathbf{D}) = \sum_{n,m} c_{n,m} \; I_1^n(\mathbf{D}) \cdot I_2^m(\mathbf{D}')$$



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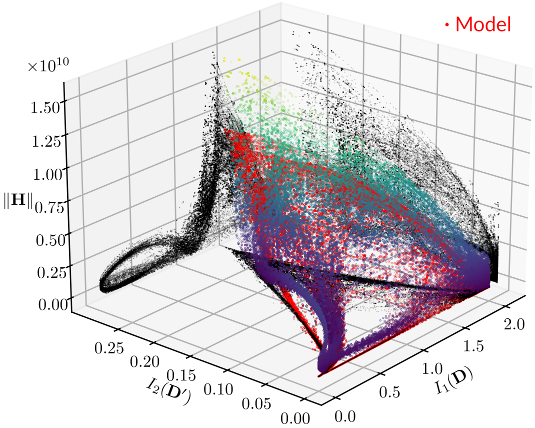

Sparse regression $(r^2 \approx 0.79) \implies H^{\mathrm{m}}(\mathbf{D}) = 18.8 \cdot 10^9 \cdot I_1^4(\mathbf{D}) \cdot I_2(\mathbf{D}')$

Summary of the state model

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$$\widetilde{\mathbf{E}}(\mathbf{D}) = 2\widetilde{\mu}(\mathbf{D})\mathbf{J} + \widetilde{\kappa}(\mathbf{D})\mathbf{1}\otimes\mathbf{1} + \frac{1}{2}\left(\widetilde{\mathbf{d}}'(\mathbf{D})\otimes\mathbf{1} + \mathbf{1}\otimes\widetilde{\mathbf{d}}'(\mathbf{D})\right) + \widetilde{\mathbf{H}}(\mathbf{D})$$

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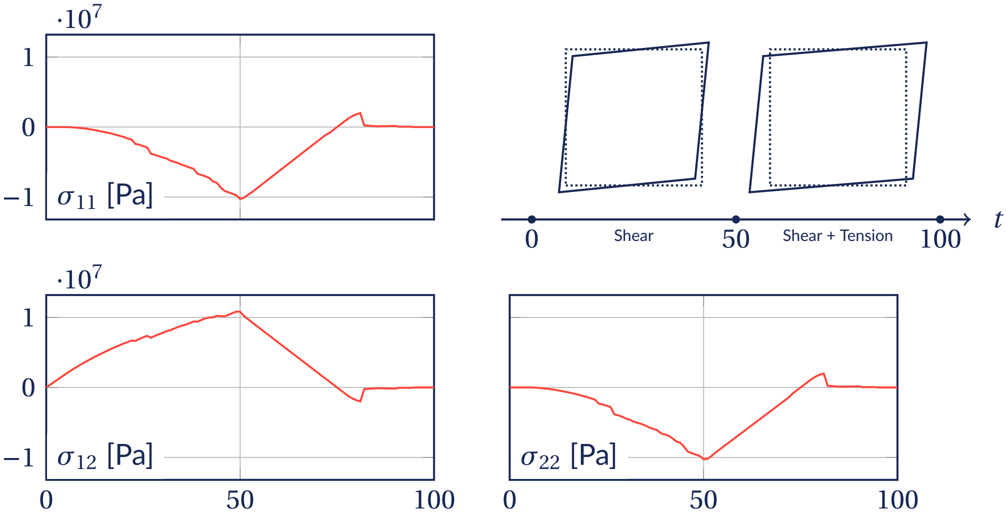
with $\mathbf{D}' * \mathbf{D}' = \mathbf{D}'\otimes\mathbf{D}' - \frac{1}{2}(\mathbf{D}':\mathbf{D}')\mathbf{J}$

Remarks

- $\widetilde{\kappa}(\mathbf{D})$ and $\widetilde{\mathbf{d}}'(\mathbf{D})$ are exact
- Parameters: μ_0, κ_0 and h

Reconstruction of stress σ from (exact) damage



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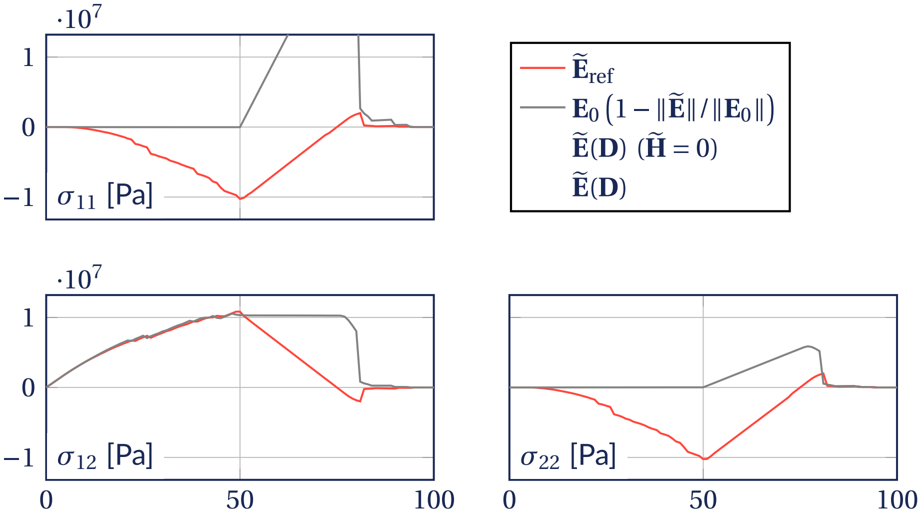

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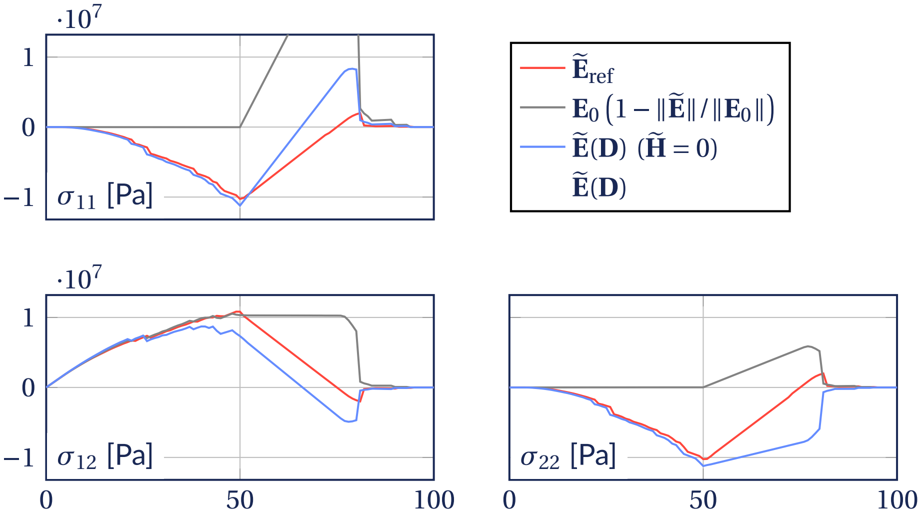

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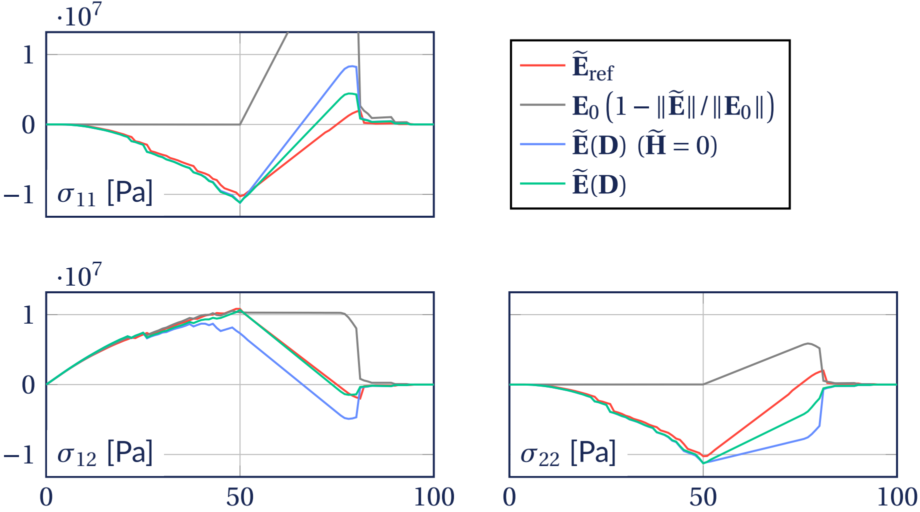
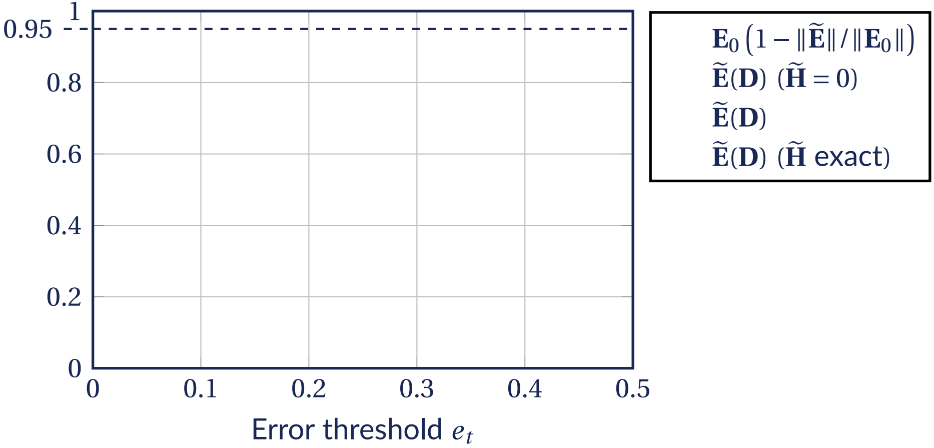


Cumulative density of error over the dataset

Proportion of $\mathbf{E}$ s.t. $\frac{\|\mathbf{E}-\mathbf{E}^m\|^2}{\|\mathbf{E}_0\|^2} \leqslant e_t$



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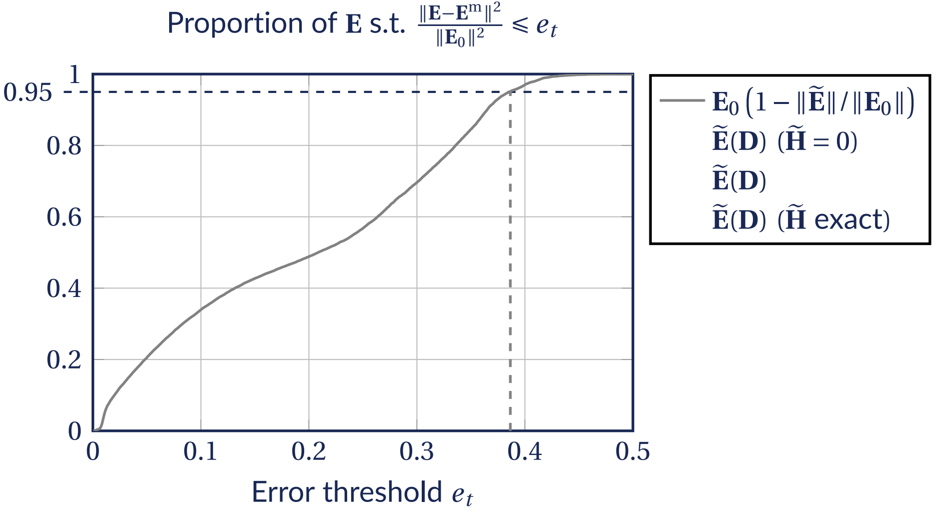

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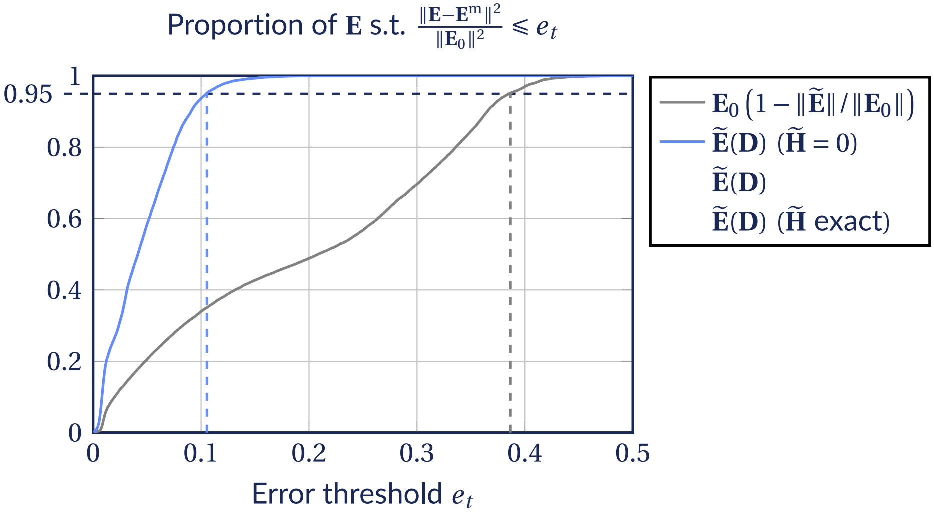

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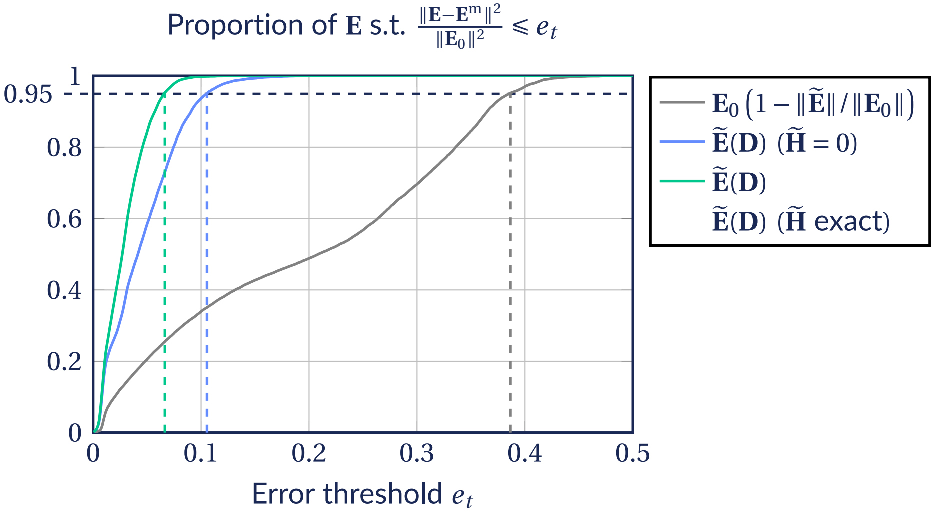

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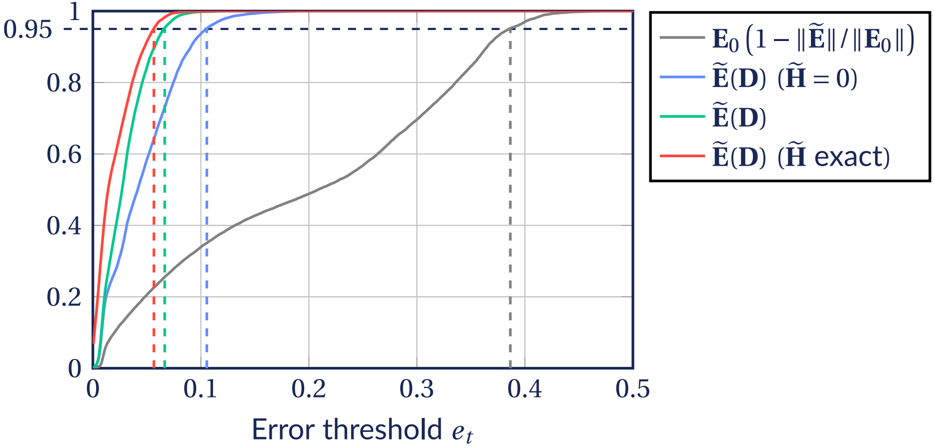

Conclusion on the state model

Proposed coupling (Loiseau et al., 2023)

$$\widetilde{\mathbb{E}}(\mathbf{D}) = 2\widetilde{\mu}(\mathbf{D})\mathbf{J} + \widetilde{\kappa}(\mathbf{D})\mathbf{1}\otimes\mathbf{1} + \frac{1}{2}\left(\widetilde{\mathbf{d}}'(\mathbf{D})\otimes\mathbf{1} + \mathbf{1}\otimes\widetilde{\mathbf{d}}'(\mathbf{D})\right) + \widetilde{\mathbf{H}}(\mathbf{D})$$

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Tools

- Distance to symmetry classes
  - Justify symmetry assumptions
- Sparse regression
  - Simplify a generic model
- Harmonic decomposition
  - Split the modelling into easier and independent modelling problems

3. Evolution law?

Objective

Describe the evolution of damage during a mechanical loading

$$\dot{\mathbf{D}} = \begin{cases} 0 & \text{if } f < 0 \text{ or } \dot{f} < 0, \\ ? & \text{otherwise.} \end{cases} \qquad f = f(\varepsilon, \mathbf{D}) \leqslant 0$$

Outline

- Presentation of the preliminary evolution law
- Application and limitations

Preliminary damage evolution model

Auxiliary damage variables

(a) $$\mathbf{D} = \mathbf{1} - (\mathbf{1} + \boldsymbol{\Delta}_a)^{-\alpha} \iff \boldsymbol{\Delta}_a = (\mathbf{1} - \mathbf{D})^{-\frac{1}{\alpha}} - \mathbf{1}$$ (Ladevèze, 1983)

(b) $$\mathbf{D} = \frac{2}{\pi} \arctan(\boldsymbol{\Delta}_b^{\alpha}) \iff \boldsymbol{\Delta}_b = \left(\tan\left(\frac{\pi}{2}\mathbf{D}\right)\right)^{\frac{1}{\alpha}}$$

Preliminary damage evolution model

Auxiliary damage variables

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Non-standard evolution law

$$\varepsilon_{\text{eq}} = \varepsilon_{\text{vM}} + k\,\text{tr}(\boldsymbol{\varepsilon})$$

$$C(\boldsymbol{\Delta}) = C_0 + S_1\,\text{tr}(\boldsymbol{\Delta}) + \frac{1}{2} S_2 \boldsymbol{\Delta}' : \boldsymbol{\Delta}'$$

Damage criterion $f(\boldsymbol{\varepsilon}, \boldsymbol{\Delta}) = \varepsilon_{\text{eq}} - C(\boldsymbol{\Delta}) \leqslant 0$

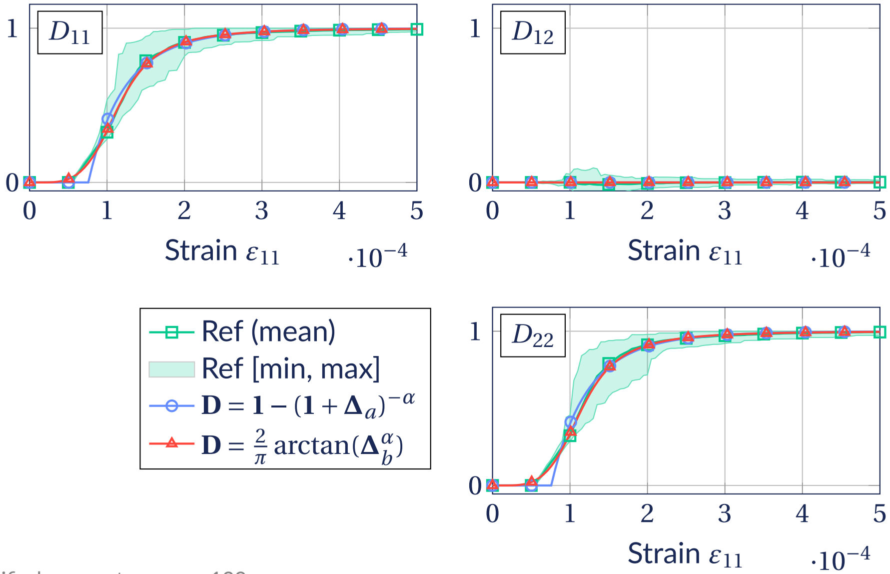
Evolution law $\dot{\boldsymbol{\Delta}} = \dot{\lambda}\mathbf{P}$

$$\dot{\lambda} = \frac{\dot{\varepsilon}_{\text{eq}}}{S_1\,\text{tr}(\mathbf{P}) + S_2 \mathbf{P}' : \boldsymbol{\Delta}'}$$

$$\mathbf{P} = \langle \boldsymbol{\varepsilon} \rangle_+ / \| \langle \boldsymbol{\varepsilon} \rangle_+ \|$$

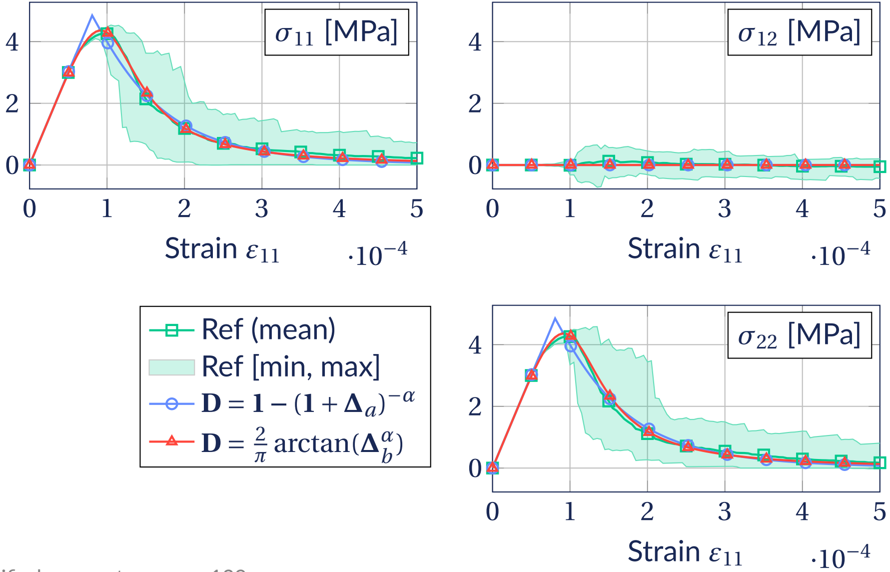
Implemented using MFront (Helfer et al., 2015) in Appendix G

Fit and illustration in bitension



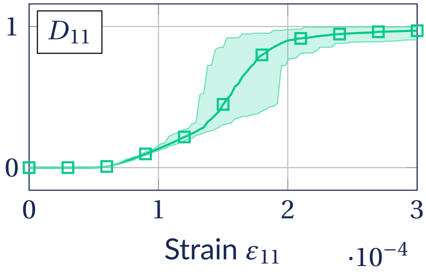
Identified parameters on p. 129

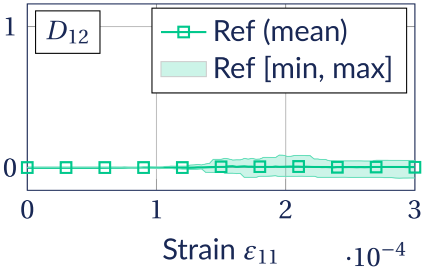
Fit and illustration in bitension



Identified parameters on p. 129

Fit and illustration in tension

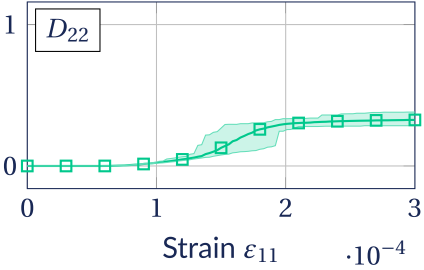




Reference $D_{22} \nearrow$

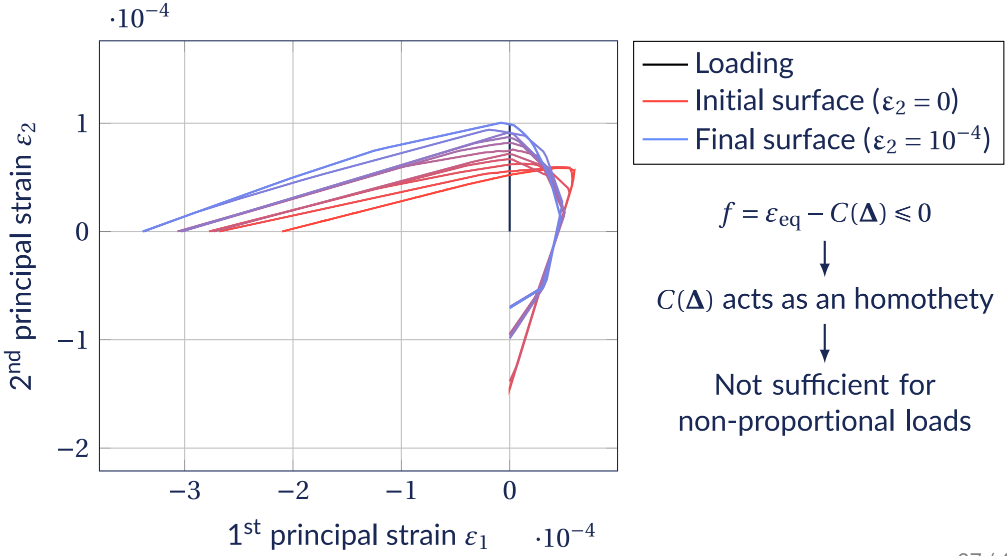
Model $\mathbf{P} \propto \langle \boldsymbol{\varepsilon} \rangle_+ = \begin{bmatrix} \varepsilon_{11} & 0 \\ 0 & 0 \end{bmatrix}$

Solution? $\mathbf{P} \propto \langle \boldsymbol{\varepsilon} \rangle_+ + I(\mathbf{D})\mathbf{1}$



Evolution of the yield surface (in tension)

Consolidation is not sufficient



$$f = \varepsilon_{eq} - C(\mathbf{\Delta}) \leqslant 0$$

↓

$C(\mathbf{\Delta})$ acts as an homothety

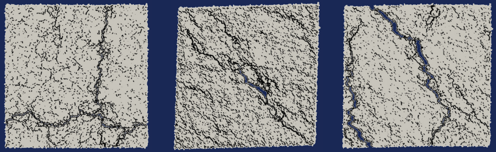
↓

Not sufficient for non-proportional loads

Conclusion

1. Virtual testing

Simulate virtual specimen with the beam-particle model to constitute the dataset of effective elasticity tensors



Conclusion

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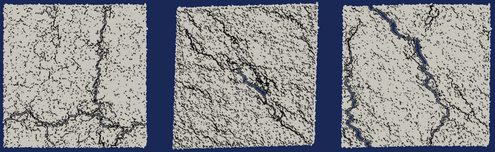


$$\widetilde{\mathbb{E}}(\mathbf{D}) = 2\widetilde{\mu}(\mathbf{D})\mathbf{J} + \widetilde{\kappa}(\mathbf{D})\mathbf{1}\otimes\mathbf{1} + \frac{1}{2}\left(\widetilde{\mathbf{d}}'(\mathbf{D})\otimes\mathbf{1} + \mathbf{1}\otimes\widetilde{\mathbf{d}}'(\mathbf{D})\right) + \widetilde{\mathbb{H}}(\mathbf{D})$$

$$\widetilde{\mu}(\mathbf{D}) = \mu_0 - \frac{\kappa_0}{4}(\operatorname{tr}\mathbf{D}) + \frac{\kappa_0 - 2\mu_0}{4}(\mathbf{D}:\mathbf{D}) \qquad \widetilde{\mathbf{d}}'(\mathbf{D}) = -2\kappa_0\mathbf{D}'$$

$$\widetilde{\kappa}(\mathbf{D}) = \kappa_0\left(1 - \frac{1}{2}\operatorname{tr}\mathbf{D}\right) \qquad \widetilde{\mathbb{H}}(\mathbf{D}) = h(\operatorname{tr}\mathbf{D})^4\mathbf{D}' * \mathbf{D}'$$

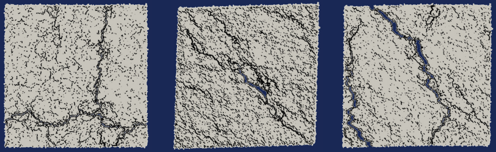
2. State model

Defined the damage variable and determined the coupling $\widetilde{\mathbb{E}}(\mathbf{D})$ between elasticity and damage

Conclusion

1. Virtual testing

Simulate virtual specimen with the beam-particle model to constitute the dataset of effective elasticity tensors



$$\widetilde{\mathbf{E}}(\mathbf{D}) = 2\widetilde{\mu}(\mathbf{D})\mathbf{J} + \widetilde{\kappa}(\mathbf{D})\mathbf{1}\otimes\mathbf{1} + \tfrac{1}{2}\left(\widetilde{\mathbf{d}}'(\mathbf{D})\otimes\mathbf{1} + \mathbf{1}\otimes\widetilde{\mathbf{d}}'(\mathbf{D})\right) + \widetilde{\mathbf{H}}(\mathbf{D})$$

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2. State model

Defined the damage variable and determined the coupling $\widetilde{\mathbf{E}}(\mathbf{D})$ between elasticity and damage

3. Evolution law

Proposed a preliminary damage evolution model and highlight its current limitations

- ✔ Use of an auxialiary damage variable
- ✖ Damaging direction
- ✖ Evolution of the yield surface

Perspectives

Enrich the model

Damage evolution

$$\dot{\mathbf{\Delta}} = \begin{cases} 0 & \text{if } f < 0, \\ \dot{\lambda}\mathbf{P} & \text{otherwise.} \end{cases}$$

- Choice of damage direction $\mathbf{P} = ?$

Other extensions

- Non-proportional loadings
  - Criterion $f(\varepsilon, \mathbf{\Delta}) = ?$
  - Crack-closure effects
- 3D formulation

Can this model fit other micro-cracked materials?

- Virtual testing
  - Another meso-scale model
- Experiments

Structural scale

- Non-local damage
  - (Pijaudier-Cabot & Bažant, 1987)
  - (Peerlings et al., 1996)
- Evolution should be formulated from the non-local damage driving quantity

Perspectives

Improve the methodology

Tools for material behavior modelling

- Relying on rigorous mathematical basis
- Using sparse and interpretable data-driven methods

Thank you for your attention!

Flavien Loiseau
Supervised by R. Desmorat, C. Oliver-Leblond

12 December 2023
Ph.D. Defense

LMPS Laboratoire de Mécanique Paris-Saclay
université PARIS-SACLAY
CentraleSupélec
école normale supérieure paris—saclay
cnrs

References I

Antonelli, A., Desmorat, B., Kolev, B., & Desmorat, R. (2022). Distance to plane elasticity orthotropy by euler–lagrange method. *Comptes Rendus. Mécanique*, *350*, 413–430. https://doi.org/10.5802/crmeca.122

Backus, G. (1970). A geometrical picture of anisotropic elastic tensors. *Reviews of Geophysics*, *8*(3), 633–671. https://doi.org/10.1029/RG008i003p00633

Berthaud, Y. (1991). Damage measurements in concrete via an ultrasonic technique. part i experiment. *Cement and Concrete Research*, *21*(1), 73–82. https://doi.org/10.1016/0008-8846(91)90033-E

Blinowski, A., Ostrowska-Maciejewska, J., & Rychlewski, J. (1996). Two-dimensional hooke's tensors - isotropic decomposition, effective symmetry criteria. *Archives of Mechanics*, *48*(2), 325–345. https://doi.org/10.24423/aom.1345

Cormery, F., & Welemane, H. (2010). A stress-based macroscopic approach for microcracks unilateral effect. *Computational Materials Science*, *47*(3), 727–738. https://doi.org/10.1016/j.commatsci.2009.10.016

References II

Cundall, P. A., & Strack, O. D. L. (1979). A discrete numerical model for granular assemblies. *Géotechnique*, *29*(1), 47–65. https://doi.org/10.1680/geot.1979.29.1.47

D'Addetta, G. A., Kun, F., & Ramm, E. (2002). On the application of a discrete model to the fracture process of cohesive granular materials. *Granular Matter*, *4*(2), 77–90. https://doi.org/10.1007/s10035-002-0103-9

Delaplace, A. (2008). *Modélisation discrète appliquée au comportement des matériaux et des structures* (Mémoire d'habilitation à diriger des recherches). Ecole Normale Supérieure de Cachan.

Delaplace, A., Pijaudier-Cabot, G., & Roux, S. (1996). Progressive damage in discrete models and consequences on continuum modelling. *Journal of the Mechanics and Physics of Solids*, *44*(1), 99–136. https://doi.org/10.1016/0022-5096(95)00062-3

Desmorat, B., & Desmorat, R. (2015). Tensorial polar decomposition of 2d fourth-order tensors. *Comptes Rendus Mécanique*, *343*(9), 471–475. https://doi.org/10.1016/j.crme.2015.07.002

References III

Desmorat, B., & Desmorat, R. (2016). Second order tensorial framework for 2d medium with open and closed cracks. *European Journal of Mechanics - A/Solids*, *58*, 262–277. https://doi.org/10.1016/j.euromechsol.2016.02.004

Desmorat, R., Gatuingt, F., & Ragueneau, F. (2007). Nonlocal anisotropic damage model and related computational aspects for quasi-brittle materials. *Engineering Fracture Mechanics*, *74*(10), 1539–1560. https://doi.org/10.1016/j.engfracmech.2006.09.012

Desmorat, R. (2016). Anisotropic damage modeling of concrete materials. *International Journal of Damage Mechanics*, *25*(6), 818–852. https://doi.org/10.1177/1056789515606509

Dormieux, L., & Kondo, D. (2016). *Micromechanics of fracture and damage* (1st ed.). Wiley.

Francfort, G. A., & Marigo, J. .-. (1998). Revisiting brittle fracture as an energy minimization problem. *Journal of the Mechanics and Physics of Solids*, *46*(8), 1319–1342. https://doi.org/10.1016/S0022-5096(98)00034-9

Grassl, P., & Jirásek, M. (2006). Damage-plastic model for concrete failure. *International Journal of Solids and Structures*, *43*(22), 7166–7196. https://doi.org/10.1016/j.ijsolstr.2006.06.032

References IV

Griffith, A. A. (1921). VI. the phenomena of rupture and flow in solids. *Philosophical Transactions of the Royal Society of London. Series A, Containing Papers of a Mathematical or Physical Character*, *221*(582), 163–198. https://doi.org/10.1098/rsta.1921.0006

Halm, D., & Dragon, A. (1996). A model of anisotropic damage by mesocrack growth; unilateral effect. *International Journal of Damage Mechanics*, *5*(4), 384–402. https://doi.org/10.1177/105678959600500403

Halm, D., & Dragon, A. (1998). An anisotropic model of damage and frictional sliding for brittle materials. *European Journal of Mechanics - A/Solids*, *17*(3), 439–460. https://doi.org/10.1016/S0997-7538(98)80054-5

Helfer, T., Michel, B., Proix, J.-M., Salvo, M., Sercombe, J., & Casella, M. (2015). Introducing the open-source mfront code generator: Application to mechanical behaviours and material knowledge management within the PLEIADES fuel element modelling platform. *Computers & Mathematics with Applications*, *70*(5), 994–1023. https://doi.org/10.1016/j.camwa.2015.06.027

References V

Herrmann, H. J., & Roux, S. (Eds.). **(1990)**. *Statistical models for the fracture of disordered media*. North-Holland. https://doi.org/10.1016/B978-0-444-88551-7.50001-X

Hrennikoff, A. (1941). Solution of problems of elasticity by the framework method. *Journal of Applied Mechanics*, *8*(4), A169–A175. https://doi.org/10.1115/1.4009129

Irwin, G. R. (1957). Analysis of stresses and strains near the end of a crack traversing a plate. *Journal of Applied Mechanics*, *24*(3), 361–364. https://doi.org/10.1115/1.4011547

Kachanov, L. M. (1958). On creep rupture time. *Izv. Acad. Nauk SSSR, Otd. Techn. Nauk*, *8*, 26–31.

Kachanov, M. (1992). Effective elastic properties of cracked solids: Critical review of some basic concepts. *Applied Mechanics Reviews*, *45*(8), 304–335. https://doi.org/10.1115/1.3119761

Krajcinovic, D. (1996). *Damage mechanics*. Elsevier.

Ladevèze, P. (1983). *Sur une théorie de l'endommagement anisotrope* (Rapport Interne No. 34). LMT Cachan.

References VI

Lehne, J., & Preston, F. (2018). *Making concrete change, innovation in low-carbon cement and concrete.* (Chatham House Report). Energy Enivronment and Resources Department: London, UK.

Lemaitre, J. (1971). *Evaluation of dissipation and damage in metals submitted to dynamic loading* (Technical).

Lemaitre, J. (1992). *A course on damage mechanics*. Springer-Verlag. https://doi.org/10.1007/978-3-662-02761-5

Lemaitre, J., & Desmorat, R. (2005). *Engineering damage mechanics: Ductile, creep, fatigue and brittle failures*. Springer-Verlag. https://doi.org/10.1007/b138882

Loiseau, F., Oliver-Leblond, C., Verbeke, T., & Desmorat, R. (2023). Anisotropic damage state modeling based on harmonic decomposition and discrete simulation of fracture. *Engineering Fracture Mechanics*, *293*, 109669. https://doi.org/10.1016/j.engfracmech.2023.109669

References VII

Lubliner, J., Oliver, J., Oller, S., & Oñate, E. (1989). A plastic-damage model for concrete. *International Journal of Solids and Structures*, *25*(3), 299–326. https://doi.org/10.1016/0020-7683(89)90050-4

Mac, M. J., Yio, M. H. N., Desbois, G., Casanova, I., Wong, H. S., & Buenfeld, N. R. (2021). 3d imaging techniques for characterising microcracks in cement-based materials. *Cement and Concrete Research*, *140*, 106309. https://doi.org/10.1016/j.cemconres.2020.106309

Mazars, J. (1984). *Application de la mécanique de l'endommagement au comportement non-linéaire et à la rupture du béton de structure* (Thèse de Doctorat d'État ès Sciences Physiques). Université Pierre et Marie Curie - Paris VI - Laboratoire de Mécanique et Technologie.

Murakami, S., & Ohno, N. (1978). A constitutive equation of creep damage in pollicristalline metals. In *IUTAM colloquium euromech*.

Murakami, S. (2012). *Continuum damage mechanics* (Vol. 185). Springer Netherlands. https://doi.org/10.1007/978-94-007-2666-6

References VIII

Oliver-Leblond, C. (2019). Discontinuous crack growth and toughening mechanisms in concrete: A numerical study based on the beam-particle approach. *Engineering Fracture Mechanics*, *207*, 1–22. https://doi.org/10.1016/j.engfracmech.2018.11.050

Oliver-Leblond, C., Desmorat, R., & Kolev, B. (2021). Continuous anisotropic damage as a twin modelling of discrete bi-dimensional fracture. *European Journal of Mechanics - A/Solids*, *89*, 104285. https://doi.org/10.1016/j.euromechsol.2021.104285

Peerlings, R. H. J., De Borst, R., Brekelmans, W. a. M., & De Vree, J. H. P. (1996). Gradient enhanced damage for quasi-brittle materials. *International Journal for Numerical Methods in Engineering*, *39*(19), 3391–3403. https://doi.org/10.1002/(SICI)1097-0207(19961015)39:19<3391::AID-NME7>3.0.CO;2-D

Pijaudier-Cabot, G., & Bažant, Z. P. (1987). Nonlocal damage theory. *Journal of Engineering Mechanics*, *113*(10), 1512–1533. https://doi.org/10.1061/(ASCE)0733-9399(1987)113:10(1512)

References IX

Ponte Castañeda, P., & Willis, J. R. (1995). The effect of spatial distribution on the effective behavior of composite materials and cracked media. *Journal of the Mechanics and Physics of Solids*, *43*(12), 1919–1951. https://doi.org/10.1016/0022-5096(95)00058-Q

Rabotnov, Y. N. (1969). Creep problems in structural members (F. A. Leckie, Ed.). *North-Holland Publishing Company, Amsterdam*. https://doi.org/10.1115/1.3408479

Rice, J. R. (1968). A path independent integral and the approximate analysis of strain concentration by notches and cracks. *Journal of Applied Mechanics*, *35*(2), 379–386. https://doi.org/10.1115/1.3601206

Richard, B., Ragueneau, F., Cremona, C., & Adelaide, L. (2010). Isotropic continuum damage mechanics for concrete under cyclic loading: Stiffness recovery, inelastic strains and frictional sliding. *Engineering Fracture Mechanics*, *77*(8), 1203–1223. https://doi.org/10.1016/j.engfracmech.2010.02.010

Rinaldi, A. (2013). Bottom-up modeling of damage in heterogeneous quasi-brittle solids. *Continuum Mechanics and Thermodynamics*, *25*(2), 359–373. https://doi.org/10.1007/s00161-012-0265-6

References X

Rinaldi, A., & Lai, Y.-C. (2007). Statistical damage theory of 2d lattices: Energetics and physical foundations of damage parameter. *International Journal of Plasticity*, *23*(10), 1796–1825. https://doi.org/10.1016/j.ijplas.2007.03.005

Terrien, M. (1980). Emission acoustique et comportement mécanique post-critique d'un béton sollicité en traction. *Bulletin de liaison des laboratoires des ponts et chaussees*, *1980*(105), 65–71.

Vakulenko, A. A., & Kachanov, M. (1971). Continuum theory of medium with cracks. *Mekhanika tverdogo tela*, *4*, 159–166.

Vannucci, P. (2005). Plane anisotropy by the polar method*. *Meccanica*, *40*(4), 437–454. https://doi.org/10.1007/s11012-005-2132-z

Vassaux, M., Oliver-Leblond, C., Richard, B., & Ragueneau, F. (2016). Beam-particle approach to model cracking and energy dissipation in concrete: Identification strategy and validation. *Cement and Concrete Composites*, *70*, 1–14. https://doi.org/10.1016/j.cemconcomp.2016.03.011

References XI

Vianello, M. (1997). An integrity basis for plane elasticity tensors. *Archives of Mechanics*, *49*(1), 197–208. https://doi.org/10.24423/aom.1401

Voyiadjis, G. Z., Taqieddin, Z. N., & Kattan, P. I. (2008). Anisotropic damage–plasticity model for concrete. *International Journal of Plasticity*, *24*(10), 1946–1965. https://doi.org/10.1016/j.ijplas.2008.04.002

Voyiadjis, G. Z., Zhou, Y., & Kattan, P. I. (2022). A new anisotropic elasto-plastic-damage model for quasi-brittle materials using strain energy equivalence. *Mechanics of Materials*, *165*, 104163. https://doi.org/10.1016/j.mechmat.2021.104163

Wriggers, P., & Moftah, S. O. (2006). Mesoscale models for concrete: Homogenisation and damage behaviour. *Finite Elements in Analysis and Design*, *42*(7), 623–636. https://doi.org/10.1016/j.finel.2005.11.008

Polynomial of invariants → linear relationship

The polynomial can be rewritten as a linear relationship

$$p(\mathbf{D}) = \begin{bmatrix} I_1\mathbf{D} & I_2\mathbf{D}' & \dots & I_1\mathbf{D}^{n_1} I_2\mathbf{D}'^{n_2} \end{bmatrix} \begin{bmatrix} c_{1,0} \\ c_{0,1} \\ \vdots \\ c_{n_1,n_2} \end{bmatrix}.$$

Remark – Numerous parameters

New question – How to fit the model?

Regression

Notations

$\mathbf{y} = y_j$	outcomes
$\mathbf{X} = (x_j)_i$	input variables
$\mathbf{c} = c_i$	coefficients

Least-square linear regression

$$\mathbf{c}^* = \arg\min_{\mathbf{c}\in\mathbb{R}^{N_c}} \left(\frac{1}{N} \|\mathbf{y} - \mathbf{X}\cdot\mathbf{c}\|_2^2 \right)$$

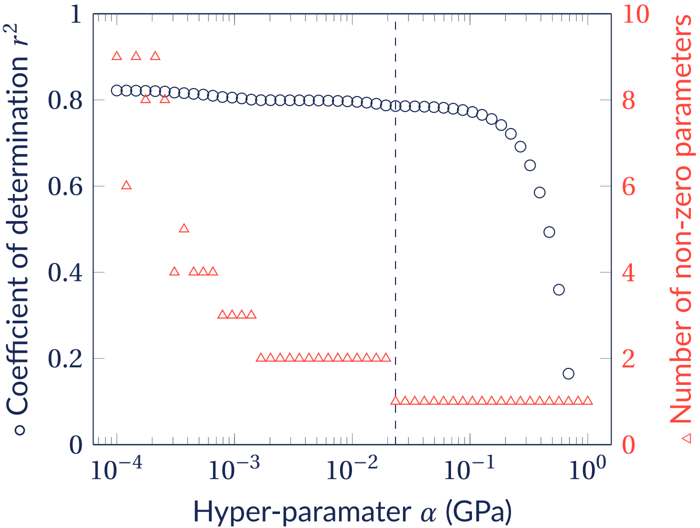
Sparse regression (LASSO)

$$\mathbf{c}^* = \arg\min_{\mathbf{c}\in\mathbb{R}^{N_c}} \left(\frac{1}{N} \|\mathbf{y} - \mathbf{X}\cdot\mathbf{c}\|_2^2 + \alpha \|\mathbf{c}\|_1 \right)$$

Features

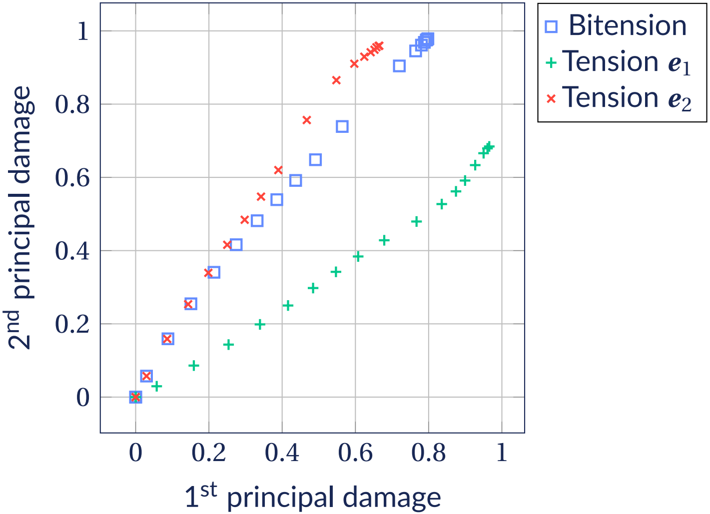
- Penalization of nonzero parameters
- Linear convex optimization problem, easy linear constraints
- Arbitrary penalization coefficient

Choosing the penalization coefficient ($n + m \leqslant 6$)



Perspective: Generic model ?

Collaboration with A. A. Basmaji (work in progress)



Measurement of the initial yield surface

Procedure

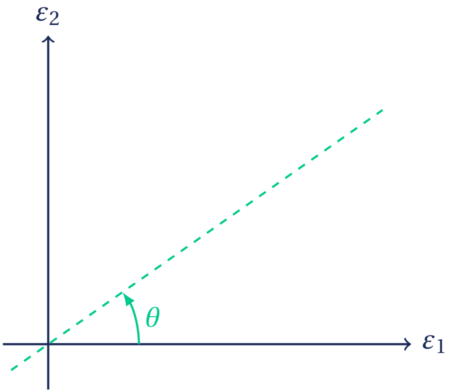
- Choose a direction θ
- Apply a loading (elastic)

$$\varepsilon_{\text{imp}} = \|\varepsilon_{\text{imp}}\| \begin{bmatrix} \cos(\theta) & 0 \\ 0 & \sin(\theta) \end{bmatrix}$$

- Get loading factor α such that the 1st beam breaks

$$\alpha = \frac{1}{f_{b^*}}, \quad f_{b^*} : \text{beam failure crit.}$$

- Calculate the yield strain $\varepsilon_y = \alpha\, \varepsilon_{\text{imp}}$



Requires 1 elastic simulation/point

Calculation of α on pp. 116–118

Measurement of the initial yield surface

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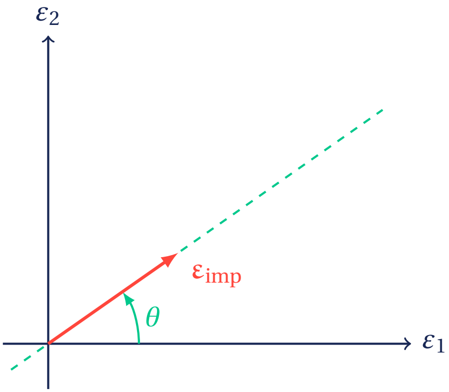

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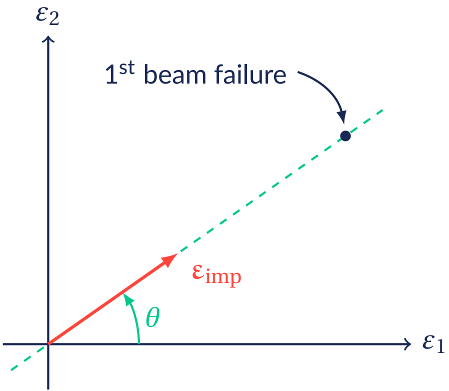

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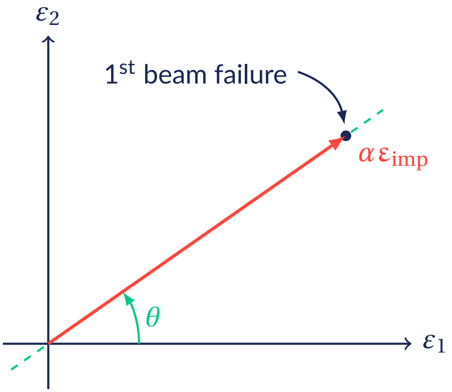


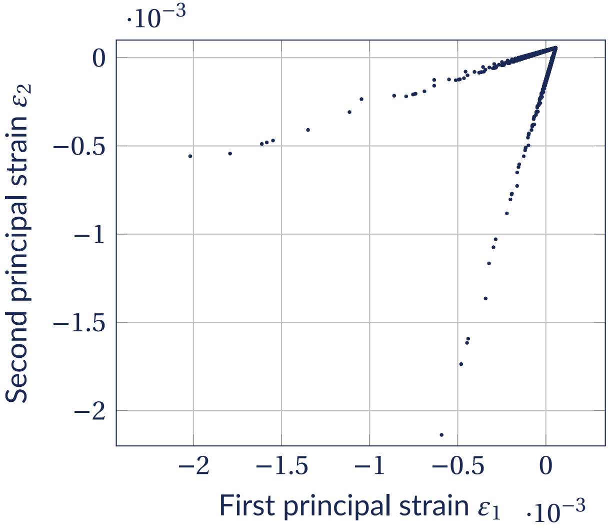
Requires 1 elastic simulation/point

Calculation of α on pp. 116–118

Initial damage criterion ($\mathbf{D} = \mathbf{0}$)

Application

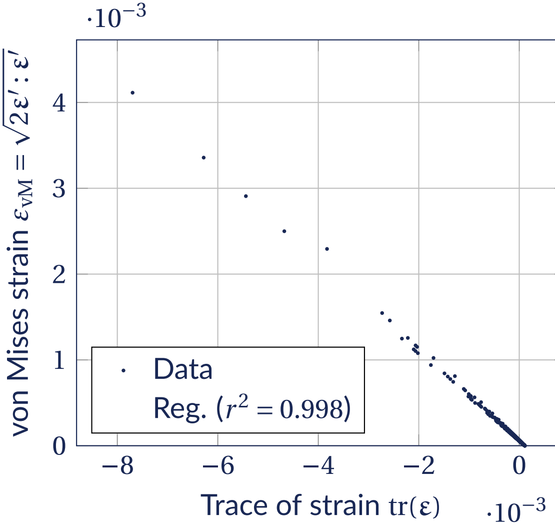
8 meso-structures



Initial damage criterion ($\mathbf{D} = \mathbf{0}$)

Application

8 meso-structures



Initial damage criterion ($\mathbf{D} = \mathbf{0}$)

Application

8 meso-structures

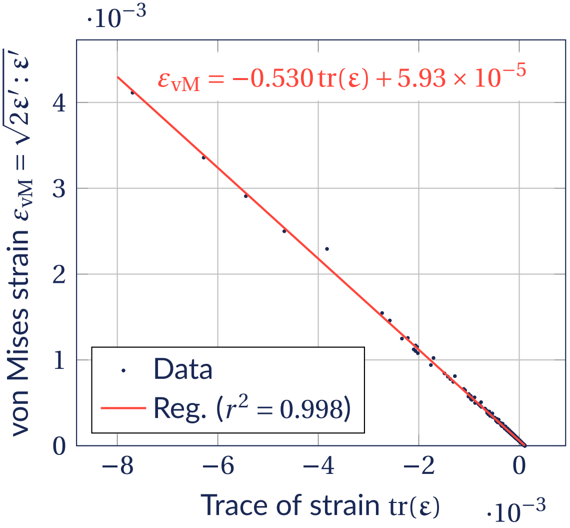
Linear regression

Damage starts when

$$\varepsilon_{vM} = -k\,\mathrm{tr}(\varepsilon) + C_0$$

where $k = 0.530$, $C_0 = 5.93 \times 10^{-5}$.
This criterion can be written

$$f(\varepsilon, \mathbf{0}) = \varepsilon_{vM} + k\,\mathrm{tr}(\varepsilon) - C_0 = 0.$$



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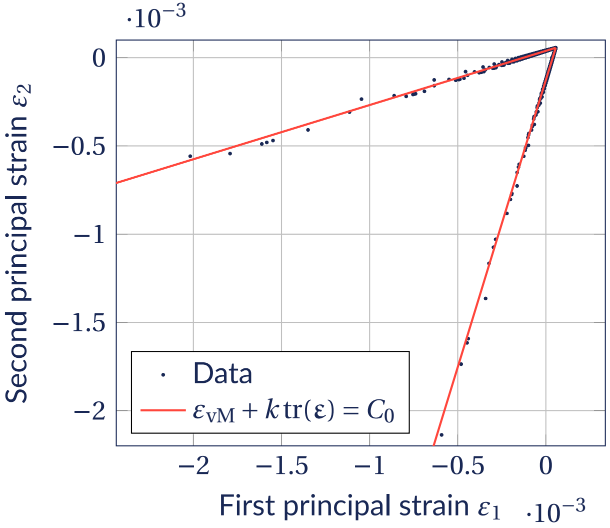


Summary of the (partial) damage evolution model

Initial damage criterion

$$f(\boldsymbol{\varepsilon}, \mathbf{D}) = \varepsilon_{\mathrm{eq}} - C(\mathbf{D})$$

where

- $\varepsilon_{\mathrm{eq}} = \varepsilon_{\mathrm{vM}} + k\,\mathrm{tr}(\boldsymbol{\varepsilon})$: equivalent strain
- $C(\mathbf{0}) = C_0$: consolidation (initial)

Non-standard damage evolution

$$\dot{\mathbf{D}} = \dot{\lambda}_{\mathbf{D}} \mathbf{P}_{\mathbf{D}}$$

where

- $\dot{\lambda}_{\mathbf{D}}$: damage multiplier
- $\mathbf{P}_{\mathbf{D}}$: damage direction (normalized)

Link between consolidation and damage evolution

$\dot{\lambda}_{\mathbf{D}}$ verifies the Kuhn-Tucker conditions

$$f \leqslant 0,\ \dot{\lambda}_{\mathbf{D}} \geqslant 0,\ f\dot{\lambda}_{\mathbf{D}} = 0 \Longrightarrow \dot{\lambda}_{\mathbf{D}} = \frac{\dot{\varepsilon}_{\mathrm{eq}}}{\mathbf{P}_{\mathbf{D}} : \frac{\partial C}{\partial \mathbf{D}}}$$

Remark

Ease bounding damage by making a change of variable

Bounding damage

Reference Mattielo, Ladeveze, + log rate of damage

Idea

Definition of an auxiliary variable $\mathbf{\Delta}$ such that $\dot{\mathbf{\Delta}} = \mathscr{G}(\mathbf{D}, \dot{\mathbf{D}})$.

In practice, we tried

$$\text{(a)} \qquad \mathbf{D} = \mathbf{1} - (\mathbf{1} + \mathbf{\Delta}_a)^{-\alpha} \iff \mathbf{\Delta}_a = (\mathbf{1} - \mathbf{D})^{-\frac{1}{\alpha}} - \mathbf{1}$$

$$\text{(b)} \qquad \mathbf{D} = \frac{2}{\pi}\arctan(\mathbf{\Delta}_b^{\alpha}) \iff \mathbf{\Delta}_b = \left(\tan\left(\frac{\pi}{2}\mathbf{D}\right)\right)^{\frac{1}{\alpha}}$$

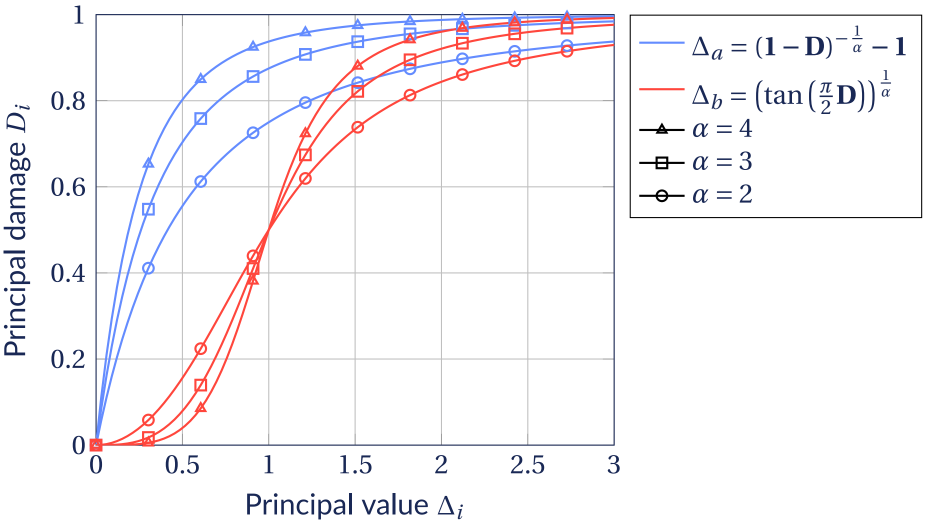
where α is the damage exponent.

Remark

Evolution of the auxiliary variable is also easier to describe (Pp. 127–128)

Function of 2nd order tensor applied on eigenvalues.

Illustration of the change of variable



Summary of the (partial) damage evolution model

Auxiliary damage variables

$$\text{(a)} \quad \mathbf{D} = \mathbf{1} - (\mathbf{1} + \boldsymbol{\Delta}_a)^{-\alpha} \iff \boldsymbol{\Delta}_a = (\mathbf{1} - \mathbf{D})^{-\frac{1}{\alpha}} - \mathbf{1}$$

$$\text{(b)} \quad \mathbf{D} = \frac{2}{\pi}\arctan(\boldsymbol{\Delta}_b^{\alpha}) \iff \boldsymbol{\Delta}_b = \left(\tan\left(\frac{\pi}{2}\mathbf{D}\right)\right)^{\frac{1}{\alpha}}$$

Damage criterion

$$f(\boldsymbol{\varepsilon}, \boldsymbol{\Delta}) = \varepsilon_{\text{eq}} - C(\boldsymbol{\Delta})$$

where

- $\varepsilon_{\text{eq}} = \varepsilon_{\text{vM}} + k\,\text{tr}(\boldsymbol{\varepsilon})$: equivalent strain
- $C(\mathbf{0}) = C_0$: consolidation (initial)

Non-standard damage evolution

$$\dot{\boldsymbol{\Delta}} = \dot{\lambda}\mathbf{P}$$

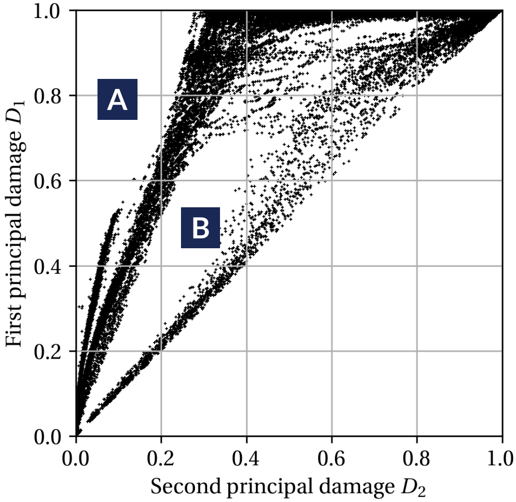
where

- $\dot{\lambda}$: auxiliary damage multiplier,
- $\mathbf{P}$: auxiliary damage direction (normalized).

Auxiliary damage multiplier from the consolidation function $\dot{\lambda} = \dot{\varepsilon}_{\text{eq}} / (\mathbf{P} : \frac{\partial C}{\partial \boldsymbol{\Delta}})$ from the Kuhn-Tucker conditions

Damaging direction

Principal damages in the dataset

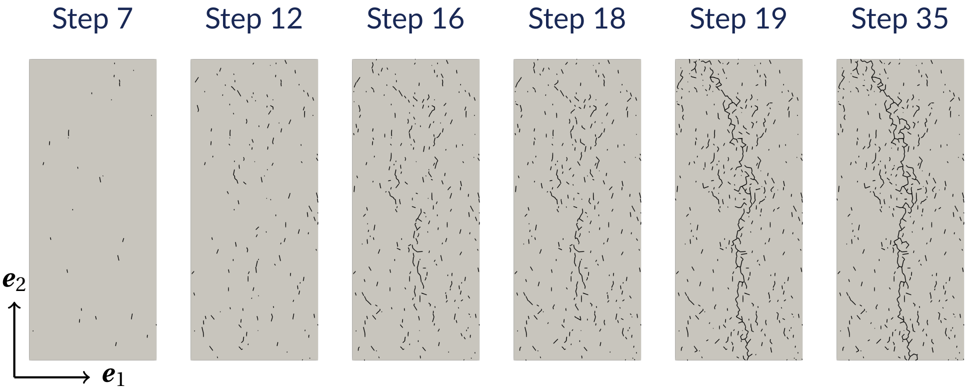


Observations

- **A** Unreachable due to damage bi-axiality
- **B** Reachable with other multi-axial loadings

Damaging direction

Bi-axiality of damage growth



Evolution of the yield surface (in bitension)

Consolidation is not sufficient